Sizing before Testing: Incentives and the Value of Pre-Experiment Information

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Firms often experiment with new initiatives before scaling them. Because experimentation is costly, however, testing every proposal may not be economically viable. They therefore often use *impact sizing*: a low-cost preliminary assessment produced through managerial judgment, predictive models, or simulations. Although managers possess valuable project-specific information, they also have considerable discretion over the estimates they report. We study how a firm should jointly design its sizing-based and experiment-based policies when a manager privately observes the sizing signal, is biased toward implementation, and may exert costly effort to improve project quality. We develop a principal–agent model and establish three main findings. First, even when report manipulation is costless, impact sizing can strictly improve the firm's payoff, but only within a wedge-shaped region of intermediate experimentation costs. When experimentation is inexpensive, the firm tests broadly and sizing is redundant; when it is too expensive, testing cannot profitably discipline exaggerated reports. Greater manipulation costs make truthful reporting easier to sustain and expand the conditions under which sizing creates value. Second, when manipulation costs alone cannot sustain the first-best allocation, truthful reporting may require policy distortions, including over-experimentation or adoption following unfavorable reports. Experimentation therefore both reveals project quality and disciplines reporting. Third, when the manager can improve project quality, experimentation also provides effort incentives. Sizing and effort can be complements when better screening raises the return to quality improvement, or substitutes when screening becomes an alternative to costly effort. Firms should therefore jointly design sizing governance, experimentation rules, and managerial effort incentives.

Key words: impact sizing; experimentation; incentive design; screening; principal-agent model

1. Introduction

Firms increasingly rely on experimentation to guide strategic and operational decisions. Large technology firms routinely use A/B tests to evaluate product changes, and mature experimentation organizations may run tens of thousands of controlled experiments each year (Kohavi et al. 2020). Experimentation at scale nevertheless requires technical infrastructure, skilled personnel, operational capacity, and sufficient user traffic (Bojinov and Gupta 2022). Moreover, most tested ideas do not improve the target outcome: in well-optimized domains such as Bing and Google, only about 10–20% of changes have a positive effect on target metrics (Kohavi et al. 2020). Firms must therefore allocate experimentation capacity selectively.

To address this constraint, some firms rely on "impact sizing": a pre-test evaluation that provides an early, albeit imperfect, estimate of a project's potential value before committing to a costly experiment (Statsig 2025). These assessments vary widely in sophistication. At one end are managers' back-of-the-envelope, self-reported estimates of a project's likely impact; at the other are elaborate simulations and predictive models. Airbnb, for example, describes a sequential process in which exploratory analysis sizes opportunities, predictive analytics identifies promising courses of action, and controlled experimentation evaluates the selected intervention (Newman 2015). At the more model-intensive end, Uber uses an agent-based marketplace simulator to evaluate new algorithms before production rollout (Chen and Wang 2019). In principle, such pre-deployment assessments can help firms prioritize which proposals advance to formal experimentation, allowing scarce testing capacity to be allocated more selectively.

Yet however sophisticated the tooling, impact sizing shares a defining feature that separates it from experimentation: its output is only as good as the assumptions and parameters the analyst supplies. A simulation does not observe the effect of a change; it computes the consequences of the inputs chosen for it. Even when those inputs are themselves estimated from data (e.g. demand elasticities, behavioral parameters, baseline rates), the analyst retains discretion over which estimates to use, which assumptions to impose, and which scenario to report, and none of these choices is disciplined by a realized outcome the way an experiment is. Because the result is assembled from the analyst's own choices rather than tested against reality, impact sizing is both far cheaper than a true test and far more discretionary, leaving ample room for optimistic inputs, selective modeling, and cherry-picking to shape the result.

Practitioners themselves are often candid about the discretion involved: in a widely upvoted r/datascience thread on how data scientists quantify the revenue impact of their work, the top response puts it bluntly:

> "The simple answer is that people make up a number. [...] Find whatever product the analysis/model is related to → find out that product's total revenue → say that the analysis/model drove that much revenue."[1]

[1] Reddit, r/datascience, "How do you all quantify the revenue impact of your work product?", 2025. `https://www.reddit.com/r/datascience/comments/1iibksg/how_do_you_all_quantify_the_revenue_impact_of/`. Accessed August 11, 2026.

Motivated by these observations, we study the fundamental tradeoff underlying impact sizing: managers have first-hand information about the projects they propose, but their incentives may differ from the firm's, and they may exploit this discretion to their own advantage. The central question is: how can a firm make impact sizing useful?

Toward this end, we develop a parsimonious principal–agent model. A firm decides whether and how to use impact sizing before undertaking costly experimentation. The manager privately observes a signal about project quality and reports it to the firm. Crucially, the manager is biased toward implementation and may misreport at a cost: the firm prefers to implement only high-quality projects, whereas the manager prefers their project to be implemented regardless of quality. This reduced-form misalignment captures settings in which implementation brings private organizational benefits, such as visibility, budget, or career advancement. When the quality of managerial information is difficult to evaluate objectively, firms may rely on subjective performance evaluation; Prendergast (1993) shows that such evaluation can induce employees to conform their reports to supervisors' views, reducing the honesty and value of internal information. Related evidence from project management shows that projected benefits may be overstated deliberately to secure approval through strategic misrepresentation or unintentionally through optimism bias (Flyvbjerg 2021).

The firm commits to a policy comprising two components: (i) a *sizing-based policy*, which maps the reported signal to a decision to directly adopt, experiment on, or reject the project, and (ii) an *experiment-based policy*, which maps the experiment outcome to an implementation decision. Experimentation is costly but perfectly informative; impact sizing is costless but imperfect. This gives experimentation a dual role: beyond acquiring information, it can *discipline* reports. We first analyze this baseline under both costly and costless manipulation.

We then extend the model to endogenous quality by allowing the manager to exert costly, unobservable effort before observing the sizing signal. The committed policy must now account jointly for reporting and effort incentives. We characterize when the firm uses sizing, induces effort, or does both.

Research questions and main findings. In this paper, we address three questions about the incentive design of impact sizing.

Q1. *If a biased manager can distort the sizing report, when is it worth using at all — and can it create value even when manipulation is costless?*

A natural conjecture is that a manipulable report is worthless. If a manager who privately observes an unfavorable signal can costlessly claim a favorable one, the sizing report looks like pure cheap talk, and one might expect the firm to ignore it. Our first result shows this intuition is incomplete: even when manipulation is costless, impact sizing can strictly improve the firm's payoff — but only within a wedge-shaped band of intermediate experimentation costs. The logic turns on experimentation's disciplining role. Testing must be costly enough that the firm would not simply experiment on every project, since otherwise the report changes no decision and is redundant; yet cheap enough that the threat of testing a favorable report remains credible, since otherwise nothing deters a low-signal manager from exaggerating. Only between these extremes can the firm use the report, and the wedge widens as the sizing signal becomes more precise. Costly manipulation reinforces this logic: raising the cost of exaggeration makes truthful reporting easier to sustain and enlarges the region in which sizing creates value. The same economic forces govern both the exogenous- and endogenous-quality settings, with the latter adding an effort-incentive margin that we take up in Q3.

Q2. *What must the firm give up to elicit truthful reports, and how does sizing precision change that cost?*

The firm has to either over-experiment or adopt unfavorable-signal projects when manipulation is insufficiently costly. The reason is the following: In the first best, which can be sustained when report manipulation is sufficiently costly, experimentation serves only information acquisition and is abandoned once its informational value falls below its cost. Under strategic reporting, experimentation has a second role: it disciplines reports. Because the manager prefers direct adoption to experimentation, which may reveal low project quality and lead to rejection, the firm can deter a low-signal manager from mimicking a high signal by attaching more experimentation to favorable reports. Consequently, the firm may experiment more aggressively than in the first best. In settings where implementation is not too costly relative to experimentation, the firm instead adopts some projects with unfavorable signals to preserve truthful reporting.

Furthermore, we find a tension between sizing precision and the cost of strategic reporting. More precise sizing raises the firm's payoff by improving screening, but it also makes the desired allocation more dependent on the signal. This sharper allocation contrast strengthens the manager's incentive to exaggerate an unfavorable report. The firm must then over-experiment after favorable reports or partially adopt after unfavorable reports to preserve truthful reporting. Consequently, both the first-best payoff and the firm's payoff under strategic reporting weakly increase with precision, while the absolute payoff gap between them weakly widens.

Q3. *When managers can also exert effort to improve project quality, how does this change the firm's use of impact sizing and experimentation?*

When managers can exert costly effort to improve project quality, experimentation takes on a third role: it motivates effort. Because only a credible commitment to test links the manager's payoff to realized quality, cutting experimentation now weakens not just screening but also the incentive to improve projects. The firm's problem thus becomes a joint choice of whether to induce effort, use sizing, use both, or neither. As in the exogenous case, sizing is valuable only in the intermediate experimentation cost wedge; what is new is that this same choice now also shapes the manager's effort — which is where sizing and effort begin to interact.

We find that sizing and effort interact in two ways. They are complements when precise sizing makes effort incentives easier to sustain: better screening raises the return to quality improvement, and higher quality raises the value of selective experimentation. In this case, sizing can induce effort in regions where the uninformative-sizing benchmark would not. They are substitutes when effort is costly and produces only limited quality improvement. In that case, the firm may rely on signal-based screening instead of inducing costly effort. Thus, analytics precision and effort incentives are jointly determined rather than independent managerial decisions.

To our knowledge, this is the first paper to formally study impact sizing, an increasingly common industry practice; analytically, it is the first to jointly design the firm's pre-experiment (sizing-based) and post-experiment (experiment-based) decision policies, so that experimentation both acquires information and disciplines the report that precedes it. In summary, the analysis yields three managerial implications. First, impact sizing should be used as a targeted screening tool, especially for projects whose experimentation costs make blanket testing unattractive but still allow selective testing to discipline reports. Second, firms should govern sizing reports, not merely collect them. The analysis identifies three levers for disciplining misreporting: raising the effective cost of manipulation, making truthful unfavorable reports less costly for the manager, and reducing the reward from favorable reports through additional experimentation. These levers help the firm use private sizing information without relying solely on costly verification. Third, firms should coordinate investments in sizing precision with incentives for quality-improving effort. More precise sizing can reinforce effort by raising the return to quality improvement, or substitute for effort by making screening more attractive.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model, including the sequence of events, the manager's reporting and effort incentives, and the firm's problems. Section 4 characterizes the optimal policy when project quality is exogenous. Section 5 extends the analysis to endogenous project quality and studies the interaction between sizing and effort incentives. Section 6 presents stylized numerical illustrations. Section 7 concludes. All technical proofs are in the Appendix.

2. Related Literature

This paper relates to three streams of literature: (i) experimentation in organizations and operations, (ii) information design, strategic communication, and scoring systems, and (iii) contracting and incentive design in operations. We discuss each in turn, highlighting how our work differs from and contributes to the existing literature.

2.1. Experimentation in Organizations and Operations

A growing literature studies how firms design and allocate experiments. Empirically, Koning et al. (2022) find that startup performance improves substantially following the adoption of A/B testing, highlighting the organizational-learning benefits of experimentation. On the theoretical side, Azevedo et al. (2020) study how scarce experimental resources should be allocated across multiple innovations, showing that the optimal balance between a few high-powered tests and many smaller tests depends on the tail behavior of innovation quality. Huang et al. (2025) show that successful changes can increase system complexity, thereby reducing the likelihood of future successful innovations and requiring more intricate and time-consuming experimental designs. A related operations management literature studies experimental design in complex environments with carryover and interference. Bojinov et al. (2023) derive optimal switchback designs under temporal carryover effects. Wager and Xu (2021) develop mean-field methods for experiments in large-scale stochastic systems with cross-unit interference, and Johari et al. (2022) analyze the resulting bias in two-sided marketplace experiments.

These studies examine experimentation primarily as a firm-controlled learning and decision tool. Some account for equilibrium responses or interference among market participants, but they do not consider a privately informed, implementation-biased manager who strategically reports a pre-experiment signal. Closest to our setting, Alonso and Câmara (2024) study an environment in which a designer selects information privately observed by an agent, the agent may misreport it at a cost, and the principal can audit the report. They show that motivating informative experimentation and deterring misreporting can be conflicting organizational goals.

Our paper instead places strategic reporting before formal experimentation. The manager's sizing report determines whether the firm directly adopts, experiments, or rejects the project, making the decision of what to test endogenous to reporting incentives. Experimentation can therefore both provide additional evidence about project quality and discipline the preceding sizing report. When project quality is endogenous, the same committed policy must also motivate the manager's hidden quality-improving effort. Our framework thus links learning, strategic reporting, and moral hazard through the joint design of the sizing-based and experiment-based rules.

2.2. Test Design and the Strategic Use of Information

Three literatures inform the strategic-reporting component of our model: test design, strategic communication, and manipulable scoring. They respectively study how evaluation technologies are designed, how private information is communicated, and how potentially distorted measures are used in decisions. Test-design models treat the precision or structure of an evaluation instrument as a design choice. Rosar (2017) studies how to design a test of an agent's quality when participation is voluntary. Perez-Richet and Skreta (2022) characterize optimal tests when an agent can covertly manipulate the test input at a cost, whereas Qiu and Shan (2025) contrast a manipulation setting, in which the agent misrepresents their type, with an investment setting, in which the agent improves their actual type. These papers make the evaluation technology itself a design choice. In our model, the sizing technology is given and its signal is privately observed by the manager. The firm's design problem is instead to commit to a report-contingent policy governing direct adoption, experimentation, and rejection, together with post-experiment implementation.

When manipulation is costless, our reporting problem resembles cheap talk (Crawford and Sobel 1982); costly manipulation connects it to models of costly misreporting (Kartik 2009). Within operations, Lu and Tomlin (2022) and Lu and Tomlin (2025) study credible communication of private social-responsibility and demand-forecast information, respectively. Our model differs by allowing the firm to commit ex ante to report-contingent experimentation before implementation. By producing additional evidence about project quality, experimentation can both inform implementation and discipline reporting.

Finally, our precision result relates to research on manipulable scoring. Ball (2025) shows that an optimal scoring rule may underweight some features to deter manipulation and overweight others so that the score remains correct on average. Frankel and Kartik (2019) show that stronger incentives can make observed actions more informative about gaming ability and less informative about natural actions. Frankel and Kartik (2022) show that committing to underuse manipulable data can attenuate information loss and improve allocation accuracy. In our model, greater sizing precision improves the firm's payoff but also weakly widens the gap from the first-best benchmark because the distortions required for truthful reporting become more costly.

2.3. Contracting and Incentive Design in Operations

Whereas the preceding subsection concerns the strategic use of information, a complementary literature studies moral hazard: how firms motivate costly, unobservable actions that generate information or improve operational outcomes; see Georgiadis (2022) for a review of theory and empirical evidence on contracting with moral hazard. Within operations, this literature examines contracts, operational controls, and information technologies as alternative instruments for motivating hidden effort and, in some settings, eliciting private information.

One stream studies contracts that combine hidden action with private information. Taylor and Xiao (2009) design contracts that induce a retailer's costly forecasting effort while eliciting the resulting private demand information for the manufacturer's production decision. Song et al.

(2024) similarly characterize salesforce contracts under both private information and unobservable actions, incorporating inventory costs and delegated ordering. A related stream uses allocation and monitoring as operational incentive instruments. Liang et al. (2022) design dynamic contracts that combine resource allocation and monetary transfers to induce effort and reduce adverse events, whereas Kim and Xu (2023) show that inspection under moral hazard can both motivate effort and correct operational errors.

Another stream examines how information technologies and human effort are jointly designed. Gurkan and de Véricourt (2022) study how contracting and data acquisition interact with a provider's hidden effort to improve a machine-learning product. Guan et al. (2025) show that a principal may optimally limit the precision of a forecasting algorithm to preserve an agent's incentive to acquire complementary private information. Dai and Taylor (2026) jointly design a generative-AI system's temperature and a success-contingent bonus when human effort is unobservable. Collectively, these studies show how data collection, algorithmic precision, and system-design choices reshape human effort incentives.

Our model brings strategic reporting and moral hazard together in a sequential screening setting. The manager first exerts unobservable effort that improves project quality, then privately observes a sizing signal, and finally may distort the report. Our incentive instrument is also distinctive: the firm offers no monetary contract and instead commits to experimentation and implementation rules that must jointly motivate effort and elicit truthful reporting. Experimentation therefore serves three linked roles: learning project quality, disciplining sizing reports, and motivating quality-improving effort. This joint design yields conditions under which sizing and effort are complements or substitutes.

3. The Model

We consider a firm (the principal) that seeks to evaluate the quality of a potential project, denoted by $q \in \{-1, 1\}$. A realization $q = 1$ corresponds to a high-quality (i.e., good) project, whereas $q = -1$ corresponds to a low-quality (i.e., bad) project.

Project quality is uncertain ex ante. Let $\gamma \in [0, 1]$ denote the prior probability of high quality, so that $\Pr(q = 1) = \gamma$ and $\Pr(q = -1) = 1 - \gamma$. If the project is implemented, the firm's payoff is q; otherwise, its payoff is zero. The manager is biased toward implementation: they obtain a normalized private benefit of one whenever the project is ultimately implemented, regardless of the realized quality q. This reduced-form payoff captures an incentive conflict that is standard in test-design models, where privately informed agents may benefit from favorable decisions even when those decisions are not optimal for the principal (Perez-Richet and Skreta 2022). In our setting, implementation itself can generate private organizational benefits, such as promotion opportunities, reputational gains, expanded span of control, budget growth, or internal visibility. Thus, the firm values implementation only when the project is high quality, whereas the manager benefits from implementation per se.

We organize the model around the timeline of the game. Section 3.1 introduces the sequence of events and the primitives associated with each step. Section 3.2 defines the payoff objects and incentive constraints. Section 3.3 formulates the firm's optimization problem.

3.1. Sequence of Events

The sequence of events is illustrated in Figure 1.

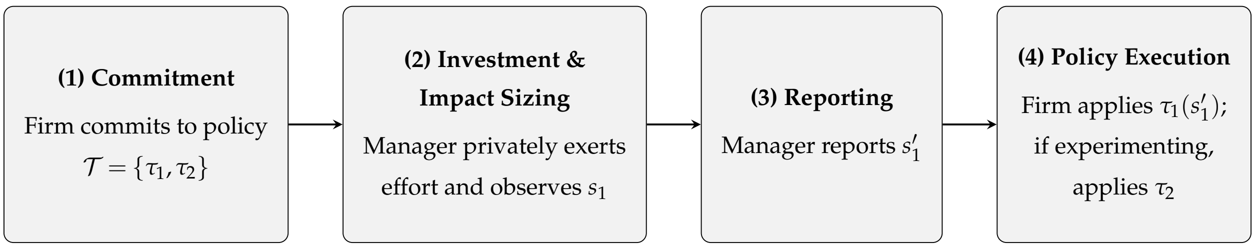


Figure 1 Timeline of the Game.

Step 1: Policy commitment. At the beginning of the game, the firm publicly commits to a sizing-based and experiment-based policy $\mathcal{T} = \{\tau_1, \tau_2\}$. The first component, i.e., the sizing-based policy, $\tau_1(s_1') = \left(\tau_1^a(s_1'), \tau_1^e(s_1'), \tau_1^r(s_1')\right)$, specifies the probabilities of three actions after receiving the manager's report $s_1' \in \{-1, 1\}$: direct adoption (a), formal experimentation (e), and immediate rejection (r). These probabilities satisfy $\tau_1^a(s_1') + \tau_1^e(s_1') + \tau_1^r(s_1') = 1$ for each $s_1' \in \{-1, 1\}$.

The second component, the *experiment-based policy* $\tau_2(s_2) \in [0, 1]$, specifies the implementation probability following outcome $s_2 \in \{-1, 1\}$. Step 4 and Appendix A.3 show that this report-independent form is without loss within the class of policies $\tau_2(s_1', s_2)$ that are monotone in s_2.

Step 2: Quality-improving effort and impact sizing. After observing the firm's committed policy $\mathcal{T}$, the manager privately chooses whether to exert costly quality-improving effort. This effort represents managerial actions that improve the underlying quality of the project, such as refining the project design, allocating attention and resources, coordinating with relevant teams, or resolving implementation risks before launch.

Without effort, the project has baseline quality prior $\gamma = \gamma_l = 1/2$, i.e., $\Pr(q = 1) = \gamma_l$. With effort, the probability that the project is high quality increases to γ_h, where $\gamma_l < \gamma_h < 1$. Thus effort raises the likelihood of a high-quality project but does not guarantee success. The cost of effort is

$$x = \beta(\gamma_h - \gamma_l), \qquad \beta \in (0, 1). \tag{1}$$

This formulation captures the idea that the cost of quality improvement scales with the magnitude of the improvement.

After the effort decision, nature draws $q \in \{-1,1\}$ according to the relevant prior, either γ_l or γ_h. The manager then conducts a costless impact-sizing exercise and privately observes signal $s_1 \in \{-1,1\}$. The precision of this signal is

$$\alpha_1 \equiv \Pr(s_1 = 1 \mid q = 1) = \Pr(s_1 = -1 \mid q = -1),$$

where $\alpha_1 > 1/2$, so that the sizing signal is informative.

For any prior $\gamma \in \{\gamma_l, \gamma_h\}$, beliefs after the sizing signal are updated according to Bayes' rule. Let Γ_1 and Γ_{-1} denote the posterior probability that the project is high quality after receiving the true sizing signal $s_1 = 1$ and $s_1 = -1$, respectively:

$$\Gamma_1(\gamma) \equiv \Pr(q = 1 \mid s_1 = 1, \gamma) = \frac{\gamma\alpha_1}{\gamma\alpha_1 + (1-\gamma)(1-\alpha_1)} \tag{2}$$

$$\Gamma_{-1}(\gamma) \equiv \Pr(q = 1 \mid s_1 = -1, \gamma) = \frac{\gamma(1-\alpha_1)}{\gamma(1-\alpha_1) + (1-\gamma)\alpha_1}. \tag{3}$$

Because $\alpha_1 > 1/2$, a positive sizing signal increases the posterior belief that the project is high quality, whereas a negative sizing signal lowers it. For compactness, define

$$\Gamma(s_1;\gamma) = \begin{cases} \Gamma_1(\gamma), & \text{if } s_1 = 1, \\ \Gamma_{-1}(\gamma), & \text{if } s_1 = -1. \end{cases} \tag{4}$$

Step 3: Reporting. After observing s_1, the manager submits a report $s_1' \in \{-1,1\}$ to the firm. We allow the manager to exaggerate a negative sizing signal, but assume that such exaggeration entails a cost. This cost captures the expected burden of sustaining an overly favorable assessment, including justification costs, audit or review risk, reputational penalties, and intrinsic costs of dishonest reporting. Such reduced-form lying or falsification costs are standard in strategic-communication and test-design models (Kartik 2009, Perez-Richet and Skreta 2022). Specifically, the manipulation cost is

$$c(s_1' \mid s_1) = \begin{cases} C, & \text{if } s_1 = -1 \text{ and } s_1' = 1, \\ 0, & \text{otherwise,} \end{cases} \quad \text{where } C \in [0,1]. \tag{5}$$

Given the committed policy $\mathcal{T}$, the manager chooses the report to maximize the expected probability that the project is ultimately implemented, net of any manipulation cost. Let $u_\gamma(s_1, s_1'; \mathcal{T})$ denote the manager's expected probability of final implementation when the quality prior is γ, the manager observes sizing signal s_1, and reports s_1'. As described in Step 4, the formal experiment perfectly reveals project quality. Hence,

$$u_\gamma(s_1, s_1'; \mathcal{T}) = \tau_1^a(s_1') + \tau_1^e(s_1') \left[\Gamma(s_1;\gamma)\tau_2(1) + \big(1 - \Gamma(s_1;\gamma)\big)\tau_2(-1)\right]. \tag{6}$$

The first term is the probability of direct adoption. The second term is the probability that the firm conducts a formal experiment, multiplied by the manager's expected probability of post-experiment implementation.

Step 4: Policy execution. After receiving report s_1', the firm applies the sizing-based policy $\tau_1(s_1')$, which determines whether the project is adopted, tested, or rejected. Experimentation costs $\kappa \geq 0$, capturing the required engineering resources, user traffic, managerial attention, and other opportunity costs. An experiment generates outcome $s_2 \in \{-1, 1\}$ with precision

$$\alpha_2 \equiv \Pr(s_2 = 1 \mid q = 1) = \Pr(s_2 = -1 \mid q = -1).$$

For tractability, the main analysis assumes perfectly informative experimentation, $\alpha_2 = 1$, so that $s_2 = q$. More generally, an experiment-based policy could condition on both the report and the experimental outcome, $\tau_2(s_1', s_2)$. Under conditional independence, Appendix A.3 shows that report dependence can be eliminated, even for an imperfect experiment, provided that the post-experiment rule is monotone in the experimental outcome.[2] Thus, report independence is without loss within the monotone class.

All exact policy characterizations are derived under $\alpha_2 = 1$, where the canonical experiment-based rule is established directly. By continuity, our main qualitative results that are supported by strict regime comparisons continue to hold when α_2 is sufficiently close to one.

3.2. Payoffs and Incentive Constraints

Incentive constraints. We define the constraints that will be used repeatedly in the firm's problem. First, the policy must be feasible. Let $\mathcal{F}$ denote the set of feasible policies:

$$\mathcal{F} = \left\{ \mathcal{T} : \begin{array}{c} \tau_1^a(s_1') + \tau_1^e(s_1') + \tau_1^r(s_1') = 1, \quad \forall s_1' \in \{-1, 1\}, \\ \tau_1^k(s_1') \in [0, 1], \quad \forall s_1' \in \{-1, 1\},\ k \in \{a, e, r\}, \\ \tau_2(s_2) \in [0, 1], \quad \forall s_2 \in \{-1, 1\}. \end{array} \right\}. \tag{7}$$

Then we define the manager's expected payoffs from not exerting and exerting quality-improving effort as $U_M^{NE}(\mathcal{T})$ and $U_M^{E}(\mathcal{T})$, respectively:

$$U_M^{NE}(\mathcal{T}) = \mathbb{E}_{s_1 \mid \gamma_l} \left[\max_{s_1' \in \{-1,1\}} \left\{ u_{\gamma_l}(s_1, s_1'; \mathcal{T}) - c(s_1' \mid s_1) \right\} \right], \tag{8}$$

$$U_M^{E}(\mathcal{T}) = \mathbb{E}_{s_1 \mid \gamma_h} \left[\max_{s_1' \in \{-1,1\}} \left\{ u_{\gamma_h}(s_1, s_1'; \mathcal{T}) - c(s_1' \mid s_1) \right\} \right] - x. \tag{9}$$

The effort incentive constraint is

$$U_M^{E}(\mathcal{T}) \geq U_M^{NE}(\mathcal{T}). \tag{EC}$$

The no-effort incentive constraint is

$$U_M^{NE}(\mathcal{T}) \geq U_M^{E}(\mathcal{T}). \tag{N-EC}$$

Following a standard tie-breaking convention in principal–agent and contract-design models, we resolve the manager's indifference in favor of the firm, as formalized below.

[2] Monotonicity requires $\tau_2(s_1', 1) \geq \tau_2(s_1', -1)$ for each report s_1'.

ASSUMPTION 1 **(Principal-preferred equilibrium selection)**. *When the manager is indifferent among multiple strategies, the equilibrium selects the strategy that gives the firm the highest expected payoff. If both the manager and the firm remain indifferent between effort and no effort, the equilibrium selects no effort. If both remain indifferent among reporting strategies conditional on the selected effort choice, truthful reporting is selected.*

Throughout the paper, all effort outcomes and regime classifications are evaluated under Assumption 1. When an effort constraint binds, we state explicitly whether the corresponding result relies on this equilibrium-selection convention.

As shown later in Lemmas 1 and 2, any optimal policy that uses the manager's sizing report can be represented, without loss of optimality, as a direct truthful-reporting policy. Conditional on the effort choice, or equivalently on the induced prior $\gamma \in \{\gamma_l, \gamma_h\}$, truthful reporting requires:

$$\left.\begin{aligned} u_\gamma(1,1;\mathcal{T}) &\geq u_\gamma(1,-1;\mathcal{T}), \\ u_\gamma(-1,-1;\mathcal{T}) &\geq u_\gamma(-1,1;\mathcal{T}) - C. \end{aligned}\right\} \tag{TR(γ)}$$

The first inequality rules out downward misreporting after a positive sizing signal. The second inequality rules out upward manipulation after a negative sizing signal.

Because truthful reporting entails no manipulation cost and implementation probabilities are nonnegative, the manager's no-effort utility is nonnegative. The effort constraint (EC) then implies nonnegative utility under effort. With the outside option normalized to zero, both participation constraints are therefore redundant.

3.3. The Firm's Problem

Given sizing signal s_1, the firm's payoff from direct adoption is

$$\pi_a(s_1;\gamma) \equiv \mathbb{E}[q \mid s_1, \gamma] = 2\Gamma(s_1;\gamma) - 1. \tag{10}$$

Under the benchmark $\alpha_2 = 1$, the formal experiment perfectly reveals quality. Thus the expected payoff from experimentation, conditional on s_1, is

$$\pi_e(s_1;\gamma,\mathcal{T}) \equiv \Gamma(s_1;\gamma)\tau_2(1) - \big(1 - \Gamma(s_1;\gamma)\big)\tau_2(-1) - \kappa, \tag{11}$$

where $\kappa \geq 0$ is the experimentation cost.

When truthful reporting is induced, the firm's expected payoff under $\mathcal{T}$ is

$$\Pi(\mathcal{T};\gamma) = \mathbb{E}_{s_1|\gamma}\left[\tau_1^a(s_1)\pi_a(s_1;\gamma) + \tau_1^e(s_1)\pi_e(s_1;\gamma,\mathcal{T})\right], \tag{12}$$

where Γ, π_a, and π_e are defined in (4), (10), and (11), respectively.

The following definition distinguishes policies according to whether they depend on the manager's report.

Definition 1 (Sizing-dependent and -independent policies). The sizing-based policy

$$\left\{\left(\tau_1^a(s_1'),\tau_1^e(s_1'),\tau_1^r(s_1')\right)\right\}_{s_1'\in\{-1,1\}}$$

is *sizing-dependent* if

$$\left(\tau_1^a(1),\tau_1^e(1),\tau_1^r(1)\right)\neq\left(\tau_1^a(-1),\tau_1^e(-1),\tau_1^r(-1)\right); \tag{SD}$$

is *sizing-independent* if

$$\left(\tau_1^a(1),\tau_1^e(1),\tau_1^r(1)\right)=\left(\tau_1^a(-1),\tau_1^e(-1),\tau_1^r(-1)\right). \tag{SInd}$$

Throughout, the firm *uses impact sizing* only when a sizing-dependent policy induces truthful reporting.

Candidate regimes. The firm chooses a feasible policy $\mathcal{T}\in\mathcal{F}$ to maximize its ex ante expected payoff, anticipating the manager's reporting and effort choices. Under Assumption 1, every policy has a uniquely selected effort outcome. We classify policies along two dimensions: whether the selected outcome involves effort, and whether the sizing-based policy depends on the manager's report. These dimensions generate the four mutually exclusive and collectively exhaustive policy regimes in Table 1.

Table 1 **Policy Regime Classification**

	Sizing-Based Policy	
Selected Effort Outcome	Sizing-Independent (SInd)	Sizing-Dependent (SD)
Effort	EI (Effort Induction Only)	DI (Dual Induced)
No effort	NI (No Induction)	SI (Sizing Induction Only)

Note. When the manager strictly prefers effort or no effort, the corresponding incentive constraint determines the outcome. When the effort constraints bind, Assumption 1 selects the outcome that gives the firm the higher expected payoff. Thus, every policy belongs to exactly one of the four regimes.

We then formulate the firm's problem separately for each regime, using the classification above. Because strict sizing dependence and principal-preferred selection can make a regime's feasible set nonclosed, we define regime values as suprema rather than maxima. Appendix A.2.3 solves the corresponding closed relaxations and classifies the resulting candidates ex post.

First, under a Dual Induction (DI) policy, the firm induces both quality-improving effort and truthful reporting of the sizing signal:

$$\begin{aligned}\Pi^{DI}=\sup_{\mathcal{T}\in\mathcal{F}}\quad & \Pi(\mathcal{T};\gamma_h)\\ \text{s.t.}\quad & (\text{TR}(\gamma))\text{ with }\gamma=\gamma_h;\ \ (\text{EC});\ \ (\text{SD}).\end{aligned} \tag{OP-DI}$$

Second, under an Effort Induction Only (EI) policy, the firm induces quality-improving effort but does not use the manager's sizing report:

$$\begin{aligned} \Pi^{EI} = \sup_{\mathcal{T}\in\mathcal{F}} \quad & \Pi(\mathcal{T};\gamma_h) \\ \text{s.t.} \quad & \text{(EC)}; \quad \text{(SInd)}. \end{aligned} \tag{OP-EI}$$

Third, under a Sizing Induction Only (SI) policy, the firm uses the manager's sizing information but does not induce quality-improving effort:

$$\begin{aligned} \Pi^{SI} = \sup_{\mathcal{T}\in\mathcal{F}} \quad & \Pi(\mathcal{T};\gamma_l) \\ \text{s.t.} \quad & (\text{TR}(\gamma)) \text{ with } \gamma=\gamma_l; \ \text{(N-EC)}; \ \text{(SD)}. \end{aligned} \tag{OP-SI}$$

Note that the no-effort constraint (N-EC) is essential: it ensures that (OP-SI) is evaluated under the baseline prior γ_l, rather than under a policy that indirectly induces quality-improving effort.

Finally, under a No Induction (NI) policy, the firm induces no quality-improving effort and does not use the sizing report. Since $\gamma_l = 1/2$, direct adoption or rejection without information has payoff zero. The firm may still conduct the formal experiment without using the sizing report:

$$\begin{aligned} \Pi^{NI} = \sup_{\mathcal{T}\in\mathcal{F}} \quad & \Pi(\mathcal{T};\gamma_l) \\ \text{s.t.} \quad & \text{(N-EC)}, \ \text{(SInd)}. \end{aligned} \tag{OP-NI}$$

The firm can always reject the project without experimenting. This all-rejection policy belongs to NI and yields zero, so $\Pi^{NI} \geq 0$.

The firm's problem is therefore

$$\Pi^* = \max_{R\in\{DI,EI,SI,NI\}} \Pi^R, \tag{13}$$

where the regime-specific payoffs are defined in (OP-DI)–(OP-NI).

4. Impact Sizing with Exogenous Quality

This section isolates the value of impact sizing when the distribution of project quality is exogenous. By removing the manager's quality-improving effort choice, we focus on the information asymmetry problem created by privately observed impact-sizing information. The relevant prior is therefore the no-effort prior $\gamma_l = 1/2$.

We first characterize a first-best benchmark (FB), in which the firm directly observes the sizing signal s_1. We then study the exogenous-quality impact-sizing problem (Ex), in which the manager privately observes s_1 and communicates it through the report s_1'. In the latter case, the firm must design its policy while accounting for the manager's reporting incentives. Throughout this section, the *FB policy* denotes the optimal policy under direct observation, whereas the *Ex policy* denotes the optimal policy under exogenous quality when the sizing signal is privately observed and must be elicited through reporting. These labels distinguish both objects from the optimal policy under endogenous quality studied in Section 5. The uninformative-sizing benchmark restricts the firm to a sizing-independent policy. Because effort is absent and $\gamma_l = 1/2$, its payoff is $\max\{1/2-\kappa, 0\}$.

4.1. First-Best with Exogenous Quality

We first derive the FB policy when the firm directly observes the sizing signal s_1. This FB policy provides a baseline for measuring the agency costs associated with strategic reporting.

PROPOSITION 1 **(FB Policy with Exogenous Quality)**. *Suppose the firm directly observes the impact-sizing signal s_1, and project quality is exogenous with prior $\gamma_l = 1/2$. In the FB benchmark, the firm adopts after a positive experimental outcome and rejects after a negative outcome, so $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. The corresponding sizing-based policy is given by:*

- *If $\kappa < 1 - \alpha_1$, the firm conducts the formal experiment after either sizing signal: $\tau_1^e(1) = \tau_1^e(-1) = 1$.*
- *If $\kappa \geq 1 - \alpha_1$, the firm adopts after a positive sizing signal and rejects after a negative sizing signal: $\tau_1^a(1) = 1$, and $\tau_1^r(-1) = 1$.*

All omitted action probabilities are zero. At $\kappa = 1 - \alpha_1$, the FB policy is not unique; the stated policy selects one FB rule.

The firm's first-best payoff is

$$\Pi^{FB} = \begin{cases} (2\alpha_1 - 1)/2, & \text{if } \kappa \geq 1 - \alpha_1, \\ 1/2 - \kappa, & \text{if } \kappa < 1 - \alpha_1. \end{cases} \tag{14}$$

Proposition 1 shows that impact sizing and formal experimentation act as substitutes in the first-best benchmark. When the experiment is inexpensive, the firm verifies every project and the sizing signal does not change the sizing-based policy. In this region, the formal experiment fully resolves quality uncertainty, so impact sizing has no incremental screening role.

When experimentation is costly, the firm uses impact sizing to economize on testing. The cutoff $\kappa = 1 - \alpha_1$ has a simple interpretation. After a positive sizing signal, experimentation improves on direct adoption only by preventing the firm from implementing a bad project, which occurs with probability $1 - \alpha_1$. After a negative sizing signal, experimentation improves on rejection only by recovering a good project, which also occurs with probability $1 - \alpha_1$. Thus the firm experiments exactly when the cost of testing is below the residual error probability of the sizing signal.

4.2. The Ex Policy: Optimal Policy with Exogenous Quality

Before characterizing the Ex policy, we establish that the firm can restrict attention to policies that induce truthful reporting.

LEMMA 1 **(Truthful Direct Implementation)**. *Fix a prior $\gamma \in \{\gamma_l, \gamma_h\}$ and an experiment-based policy τ_2. For any sizing-based policy τ_1 and any reporting strategy $\sigma : S_1 \to \Delta(S_1)$ selected under Assumption 1, there exists a direct sizing-based policy $\tilde{\tau}_1$ under which truthful reporting is selected and the firm obtains the same signal-contingent allocation, and hence the same payoff.*

By Lemma 1, the firm can restrict attention, without loss of generality, to direct policies under truthful reporting. Hence, the firm's problem under exogenous quality is given by:

$$\begin{aligned} \Pi^{\mathrm{Ex}} = \max_{\mathcal{T} \in \mathcal{F}} \quad & \Pi(\mathcal{T}; \gamma_l) \\ \text{s.t.} \quad & (\mathrm{TR}(\gamma)) \text{ with } \gamma = \gamma_l. \end{aligned} \tag{OP-Ex}$$

Here, $\Pi(\mathcal{T}; \gamma_l)$ is defined in (12). We call an optimal solution $\mathcal{T}^{\mathrm{Ex}}$ to Problem (OP-Ex) an Ex policy; the superscript Ex identifies its allocation probabilities and payoff. No separate participation constraints are required because truthful reporting entails zero manipulation cost and gives the manager a nonnegative expected implementation benefit.

Solving Problem (OP-Ex) yields the following result.

PROPOSITION 2 **(Characterization of the Ex Policy)**. *Suppose project quality is exogenous with prior $\gamma_l = 1/2$. An Ex policy $\mathcal{T}^{\mathrm{Ex}}$ can be chosen so that its experiment-based component satisfies*

$$\tau_2^{\mathrm{Ex}}(1) = 1, \qquad \tau_2^{\mathrm{Ex}}(-1) = 0.$$

The sizing-based component of $\mathcal{T}^{\mathrm{Ex}}$ is given in Table 2. All omitted rejection probabilities are determined by feasibility: $\tau_1^{r,\mathrm{Ex}}(s) = 1 - \tau_1^{a,\mathrm{Ex}}(s) - \tau_1^{e,\mathrm{Ex}}(s)$, $s \in \{-1, 1\}$.

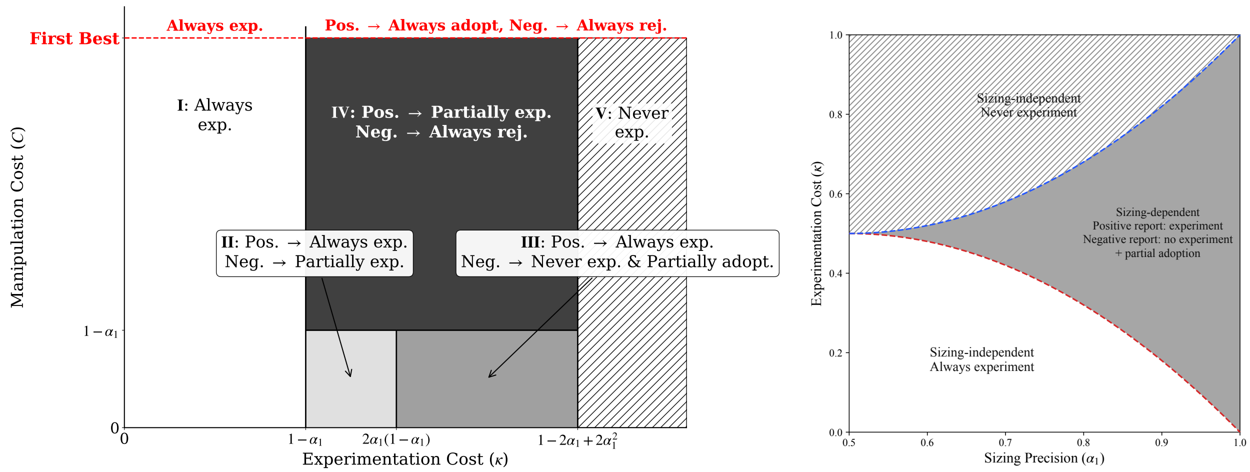
Table 2 Sizing-Based Component of the Ex Policy

Region	Conditions	$\left(\tau_1^{a,\mathrm{Ex}}(1), \tau_1^{e,\mathrm{Ex}}(1), \tau_1^{a,\mathrm{Ex}}(-1), \tau_1^{e,\mathrm{Ex}}(-1)\right)$
I	$\kappa \le 1 - \alpha_1$	$(0, 1, 0, 1)$
II	$\kappa \in (1 - \alpha_1, 2\alpha_1(1 - \alpha_1)]$, $C \le 1 - \alpha_1$	$\left(0, 1, 0, 1 - \frac{C}{1-\alpha_1}\right)$
III	$\kappa \in (2\alpha_1(1 - \alpha_1), 1 - 2\alpha_1 + 2\alpha_1^2]$, $C \le 1 - \alpha_1$	$(0, 1, 1 - \alpha_1 - C, 0)$
IV	$\kappa \in (1 - \alpha_1, 1 - 2\alpha_1 + 2\alpha_1^2]$, $C > 1 - \alpha_1$	$\left(1 - \frac{1-C}{\alpha_1}, \frac{1-C}{\alpha_1}, 0, 0\right)$
V	$\kappa > 1 - 2\alpha_1 + 2\alpha_1^2$	$(1, 0, 1 - C, 0)$ or $(C, 0, 0, 0)$

Figure 2(a) illustrates how the Ex policy varies with experimentation cost κ and manipulation cost C. When experimentation is inexpensive, the firm experiments after both reports. As κ increases, it relies more on sizing, moving toward adoption after positive reports and rejection after negative reports. Reporting incentives, however, can distort this transition.

In Region III, experimentation is relatively costly and manipulation is inexpensive. The firm then partially adopts after a negative report, making truthful disclosure of an unfavorable signal less punitive and reducing the manager's incentive to exaggerate. This concession sacrifices allocative efficiency but avoids more costly experimentation.

When $C > 1 - \alpha_1$, manipulation itself sufficiently discourages exaggeration, so the firm can reject negative reports while mixing direct adoption and experimentation after positive reports.



(a) Ex policy varying with κ and C

(b) $C = 0$

Figure 2 The Ex Policy under Impact Sizing with Exogenous Quality

More broadly, the firm can discipline reporting by increasing manipulation costs, making negative reports less punitive, or subjecting positive reports to additional experimentation. The Ex policy selects among these instruments according to their relative costs.

4.3. Distortions and the Value of the Ex Policy

We compare the Ex policy in Proposition 2 with the FB benchmark in Proposition 1. This comparison isolates the distortions caused by strategic reporting.

COROLLARY 1 **(Distortions Relative to FB)**. *Relative to the FB policy, the Ex policy has the following properties.*

1. ***Weak over-experimentation.*** *For each sizing signal $s \in \{-1, 1\}$, the firm experiments weakly more often under strategic reporting than under the first best: $\tau_1^{e,\text{Ex}}(s) \geq \tau_1^{e,\text{FB}}(s)$. The inequality is strict after a positive report in the intermediate-cost regions whenever $C < 1$.*

2. ***First-best implementation with prohibitive manipulation.*** *If $C = 1$, the firm attains a first-best allocation and payoff.*

3. ***Divergent no-experimentation thresholds.*** *Under the first best, a no-experimentation policy is optimal whenever $\kappa \geq 1 - \alpha_1$, and this cutoff decreases in α_1. Under strategic reporting with $C < 1$, a no-experimentation policy is optimal only when $\kappa \geq 1 - 2\alpha_1 + 2\alpha_1^2$, and this cutoff increases in α_1.*

4. ***Increasing agency cost of precision.*** *For fixed κ and C, the agency cost $\Pi^{\text{FB}} - \Pi^{\text{Ex}}$ is weakly increasing in α_1. It is identically zero when $C = 1$.*

Corollary 1 shows that strategic reporting changes the role of experimentation. In the FB benchmark, experimentation is used only to learn project quality. Once the experimentation cost exceeds the learning benefit, the firm stops experimenting and relies on the observed sizing signal. Under strategic reporting, experimentation also serves an *incentive role*. A manager with a negative sizing signal prefers to be treated as if the signal were positive, because positive reports lead to a higher implementation probability. By increasing the probability of experimentation after favorable reports, the firm reduces the attractiveness of such exaggeration. The cost is over-experimentation relative to the FB benchmark.

When manipulation is prohibitively costly, $C = 1$, the reporting friction disappears and the firm implements the FB allocation. When $C < 1$, the experimentation cutoff moves differently under strategic reporting. In the FB benchmark, the cutoff $1 - \alpha_1$ decreases with signal precision because more precise sizing information substitutes for formal experimentation. Under strategic reporting, the no-experimentation cutoff $1 - 2\alpha_1 + 2\alpha_1^2$ increases with precision. The reason is that a more precise sizing signal increases the implementation-payoff gap between being treated as a positive-report type and being treated as a negative-report type. As α_1 increases, the firm would like to adopt positive-signal projects more aggressively and reject negative-signal projects more aggressively. This sharper allocation contrast increases the gain from exaggeration for a manager with a negative signal. To preserve truthful reporting, the firm must dampen the reward from a positive report, for example by replacing some direct adoption with experimentation. Because experimentation may reveal low quality and lead to rejection, it is less attractive to a biased manager than direct adoption.

The agency cost of strategic reporting weakly increases with signal precision because a more precise signal makes the firm's desired decisions more report-contingent. This increases the manager's gain from exaggerating an unfavorable signal. To sustain truthful reporting, the firm must narrow the implementation-payoff difference across reports, either by replacing direct adoption after a favorable report with experimentation or by partially adopting after an unfavorable report. Both distortions become more costly as precision increases because favorable-signal projects become more valuable and unfavorable-signal projects less valuable. Thus, although precision raises the FB payoff, it also increases the cost of truthful elicitation, widening the agency-cost gap.

We conclude this section by examining the payoff from impact sizing.

COROLLARY 2 **(Payoff from Impact Sizing with Exogenous Quality)**. *Under exogenous quality, the Ex payoff Π^{Ex} has the following properties:*

1. ***Value of sizing.*** *Impact sizing is weakly beneficial relative to the uninformative-sizing benchmark:*

$$\Pi^{\text{Ex}} \geq \max\{1/2 - \kappa, 0\}. \tag{15}$$

2. ***When sizing creates no value.*** *The inequality in* (15) *holds with equality in the following cases:*

- *If $C = 0$, impact sizing creates no payoff gain when experimentation is either sufficiently cheap or sufficiently costly. Specifically, when $\kappa \le 2\alpha_1(1-\alpha_1)$, the firm experiments after both reports; when $\kappa \ge 1 - 2\alpha_1 + 2\alpha_1^2$, an Ex policy can be chosen not to experiment, and the firm is indifferent between adopting and rejecting all projects.*
- *If $C > 0$, impact sizing creates no payoff gain when experimentation is sufficiently cheap, $\kappa \le 1 - \alpha_1$. In this case, the firm experiments after both reports.*

3. ***Comparative statics.*** *The payoff Π^{Ex} is weakly increasing in the sizing precision α_1 and the manipulation cost C, and weakly decreasing in the experimentation cost κ.*

Corollary 2 shows that impact sizing is weakly beneficial because the firm can always ignore the report. It creates a strict payoff gain only when the report changes the firm's adoption, experimentation, or rejection decision.

A positive manipulation cost broadens the conditions under which impact sizing can create value because it directly discourages exaggeration. When experimentation is inexpensive, however, the firm still experiments after both reports, so sizing adds no value. As experimentation becomes more costly, a positive manipulation cost can support report-contingent decisions with less reliance on testing.

The costless-manipulation case, $C = 0$, produces a sharper pattern, illustrated in Figure 2(b). When experimentation is inexpensive, the firm experiments after both reports. When it is sufficiently costly, the firm cannot profitably use experimentation to discipline reporting and therefore neither experiments nor uses the sizing report. Only at intermediate experimentation costs does impact sizing create a strict payoff gain. In this region, the firm experiments after a favorable report and partially adopts after an unfavorable report to sustain truthful reporting. Greater sizing precision makes such report-based screening more valuable.

The next section introduces unobservable quality-improving effort while retaining the costless-manipulation benchmark.

5. Impact Sizing with Endogenous Quality

Section 4 treats the distribution of project quality as exogenous. We now allow the manager to incur cost $x = \beta(\gamma_h - \gamma_l)$, as defined in (1), to raise the probability of high quality from γ_l to γ_h. This unobservable investment captures pre-launch activities such as market research, prototyping, and feasibility analysis. The firm's committed policy must now both motivate this investment and elicit truthful reports of the manager's private sizing signal.

We focus on the costless-manipulation case, $C = 0$. This is the most stringent environment for impact sizing because the manager can misreport the sizing signal without direct penalty. It also keeps the adverse-selection friction from Section 4 active, allowing us to isolate how effort incentives change the value and use of sizing. With endogenous quality, the firm chooses among four regimes defined in (OP-DI)–(OP-NI): EI, SI, DI, and NI. These regimes differ in whether the firm induces quality-improving effort and whether it uses the sizing report.

The analysis proceeds as follows. We first characterize the first-best benchmark with endogenous quality in Proposition 3. We then solve the firm's policy problem by comparing EI, SI, DI, and NI, leading to the optimal policy characterization in Proposition 4. The remaining results study the structure of this optimum. Proposition 5 shows that impact sizing is used in a wedge-shaped intermediate-cost region, and Corollary 3 identifies a minimum precision level below which sizing has no payoff value. Finally, Propositions 6 and 7 show that sizing can either complement or substitute for effort. Thus, the endogenous-quality setting highlights that sizing is not only an information-acquisition tool; it also changes the firm's incentive to induce quality-improving effort.

5.1. First-Best with Endogenous Quality

We begin with the first-best benchmark, in which the firm directly observes the sizing signal s_1. The following definition makes the treatment of effort cost explicit.

DEFINITION 2 (FIRST-BEST BENCHMARK). In the first-best benchmark, the sizing signal and quality-improving effort are observable and contractible. The decision maker conditions the sizing-based policy directly on s_1 and chooses whether effort is exerted to maximize the project payoff net of effort cost:

$$\max\left\{\max_{\mathcal{T}\in\mathcal{F}}\Pi(\mathcal{T};\gamma_l),\quad \max_{\mathcal{T}\in\mathcal{F}}\left[\Pi(\mathcal{T};\gamma_h)-x\right]\right\}. \tag{FB}$$

The first term corresponds to no effort and the second to effort. We refer to an optimizer of (FB) as the *first-best policy*.

This benchmark removes both reporting and effort-incentive frictions and isolates the efficient use of effort and experimentation.

PROPOSITION 3 **(First-Best Policy with Endogenous Quality)**. *Under the first-best benchmark in Definition 2, quality-improving effort is always exerted, so the relevant prior is γ_h. The optimal experiment-based policy is $\tau_2(1)=1$ and $\tau_2(-1)=0$. The optimal sizing-based policy is characterized by two cutoffs:*

$$\underline{\kappa}=1-\Gamma(1;\gamma_h),\qquad \overline{\kappa}=\begin{cases}1-\Gamma(-1;\gamma_h), & \text{if } \gamma_h\geq\alpha_1,\\ \Gamma(-1;\gamma_h), & \text{if } \gamma_h<\alpha_1.\end{cases}$$

Here, $\Gamma(\cdot)$ is defined in (4). *Specifically:*

- *If $\kappa<\underline{\kappa}$, the firm experiments after both sizing signals.*

- *If $\underline{\kappa} \leq \kappa < \overline{\kappa}$, the firm adopts directly after a positive sizing signal and experiments after a negative sizing signal.*
- *If $\kappa \geq \overline{\kappa}$, the firm adopts directly after a positive sizing signal. After a negative sizing signal, it adopts directly if $\gamma_h \geq \alpha_1$, and rejects if $\gamma_h < \alpha_1$.*

At the cutoff values, multiple policies may be optimal.

Proposition 3 separates the efficient use of quality-improving effort from the efficient use of experimentation. In the first-best benchmark, effort increases the maximal expected project payoff by at least $\gamma_h - \gamma_l$, while its cost is $\beta(\gamma_h - \gamma_l)$. Because $\beta < 1$, the net gain from effort is at least $(1-\beta)(\gamma_h - \gamma_l) > 0$. Effort is therefore always chosen in the first-best policy.

Given effort, the experimentation decision is signal-contingent. After a sizing signal, the firm compares the cost of experimentation with the value of resolving the remaining quality uncertainty. When the posterior belief is above $1/2$, direct adoption is the default action. Experimentation is then valuable because it prevents the firm from adopting bad projects. The gain from experimentation equals the posterior probability that the project is bad, so the firm experiments only if this probability exceeds κ.

When the posterior belief after a negative sizing signal is below $1/2$, rejection is the default action. Experimentation then serves a different role: it identifies good projects that would otherwise be rejected. The gain from experimentation equals the posterior probability that the project is good, so the firm experiments only if this probability exceeds κ. Thus the two cutoffs in Proposition 3 reflect the same principle: the firm experiments exactly when the value of correcting the default action exceeds the experimentation cost.

5.2. Optimal Policy under Endogenous Quality

Before characterizing the optimal policy, we establish that the firm can restrict attention to policies that simultaneously induce truthful reporting and a definite effort level, without loss of generality within each instrument combination.

LEMMA 2 **(Truthful Direct Implementation with Endogenous Quality)**. *Consider the endogenous-quality setting with costless manipulation (i.e., $C = 0$). Fix a policy $\mathcal{T} = (\tau_1, \tau_2)$, and suppose that under $\mathcal{T}$, Assumption 1 selects effort-induced prior $\gamma^* \in \{\gamma_l, \gamma_h\}$ and reporting strategy $\sigma : S_1 \to \Delta(S_1)$. Then there exists a direct policy $\tilde{\mathcal{T}} = (\tilde{\tau}_1, \tau_2)$, under which the same effort choice is selected, truthful reporting is selected, and the firm obtains the same equilibrium payoff.*

With Lemma 2, the firm's optimization reduces to four regimes: DI induces effort and uses sizing, EI induces effort only, SI uses sizing only, and NI induces neither. This decomposition is exhaustive because both effort and sizing utilization are binary decisions, and truthful direct implementation entails no loss whenever sizing is used. We obtain a closed-form characterization of the candidate policies within each regime. Appendix A.2.3 derives their policy rules, payoffs, feasibility conditions, and classifications under Assumption 1. The following proposition summarizes the resulting comparison.

PROPOSITION 4 **(Finite-Candidate Characterization of the Optimal Policy)**. *Suppose project quality is endogenous and $C = 0$. There exists an optimal policy whose experiment-based rule satisfies $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. Its payoff is*

$$\begin{aligned} \Pi^* &= \max\left\{\Pi_1^{EI}, \Pi_2^{EI}, \max_{j\in\{1,2,3\}} \Pi_j^{DI}, \max_{j\in\{1,2,3,4\}} \Pi_j^{SI}, \Pi^{NI}\right\} \\ &= \max\left\{\Pi_1^{EI}, \Pi_2^{EI}, \max_{j\in\{1,2,3\}} \Pi_j^{DI}, \max_{j\in\{1,2,3,4\}} \Pi_j^{SI}, 0\right\}, \end{aligned} \tag{16}$$

where $\Pi^{NI} = \beta \max\{1/2 - \kappa, 0\}$. A DI or SI candidate is included only when it is feasible and Assumption 1 assigns its selected effort and reporting strategies to the stated regime. The second equality follows because the positive branch of Π^{NI} is strictly dominated by EI–2, whereas its zero branch is attained by rejection. An optimal policy is any retained candidate attaining the displayed maximum.

Appendix A.2.3 provides the closed-form candidate policies and payoffs and proves both the candidate enumeration and the NI dominance result.

Figure 3 plots the optimal regime in the (α_1, κ) plane for two parameter configurations. Although the boundaries vary with the effort-cost coefficient β and effort effectiveness γ_h, both panels illustrate the same qualitative structure.

Effort-inducing regimes, EI and DI, arise when experimentation is sufficiently inexpensive. Experimentation makes implementation contingent on project quality and thereby rewards quality-improving effort. As κ increases, providing this incentive becomes more costly, and the firm eventually switches to a no-effort regime.

Within the sizing-dependent regions, the optimal policy retains a reporting distortion identified under exogenous quality: the firm may partially adopt after an unfavorable report. This raises the manager's payoff from reporting an unfavorable signal truthfully and thereby reduces the incentive to exaggerate. Appendix A.2.3 provides the corresponding policy characterization.

Turning from the form of the policy to when sizing is used, the figure suggests a wedge-shaped sizing region. When experimentation is inexpensive, the firm tests broadly and gains little from sizing. When experimentation is too costly, testing cannot profitably discipline reporting. Between these extremes, sizing allows the firm to use experimentation selectively, and this region expands as the sizing signal becomes more precise. Proposition 5 formalizes these patterns.

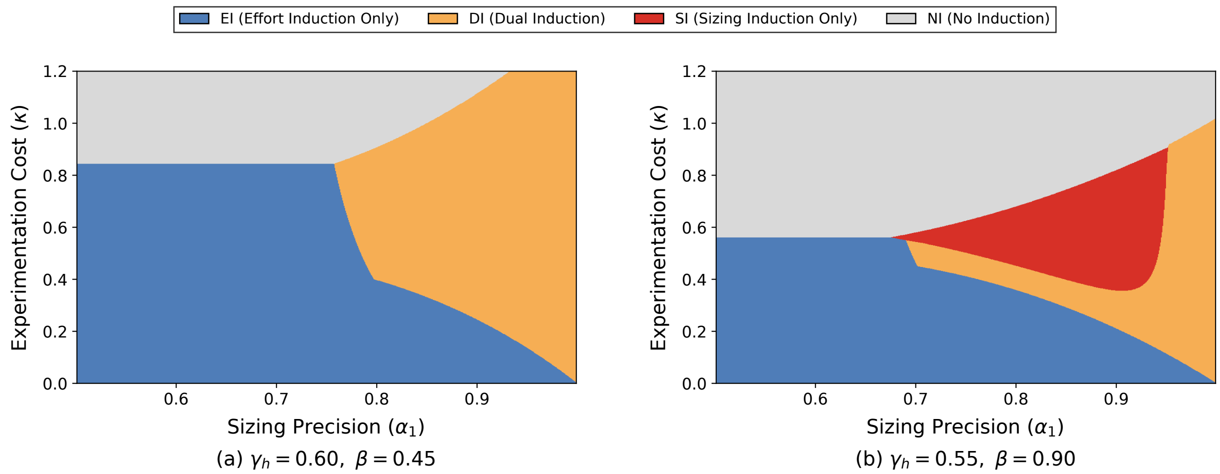


Figure 3 **Optimal Sizing-Based Policy under Endogenous Quality**

5.3. Features of the Optimal Policy

The preceding characterization suggests that impact sizing is valuable only over an intermediate range of experimentation costs. We formalize this pattern by comparing the best sizing-dependent policy with the best sizing-independent alternative. Let $\Pi^{SD}(\alpha_1,\kappa)$ denote the supremum payoff over feasible policies classified as DI or SI, and define

$$\Pi^{IND}(\kappa) = \max\{\Pi_1^{EI}, \Pi_2^{EI}, \Pi^{NI}\} = \max\{\Pi_1^{EI}, \Pi_2^{EI}, 0\},$$

where the second equality follows from (A.18). We say that impact sizing is *strictly optimal* when

$$\Pi^{SD}(\alpha_1,\kappa) > \Pi^{IND}(\kappa).$$

PROPOSITION 5 **(Optimal Use of Impact Sizing)**. *Fix* (β,γ_h) *with* $\beta \in (0,1)$ *and* $\gamma_h \in (1/2,1)$. *Suppressing the dependence on these fixed parameters, there exist boundary functions* $K_L(\alpha_1)$ *and* $K_U(\alpha_1)$ *such that impact sizing is strictly optimal if and only if*

$$K_L(\alpha_1) < \kappa < K_U(\alpha_1).$$

The boundaries coincide when the impact-sizing region is empty. In particular, impact sizing is not strictly optimal when $\alpha_1 \le \gamma_h$.

Moreover:

1. $K_L(\alpha_1)$ *is weakly decreasing in* α_1, *whereas* $K_U(\alpha_1)$ *is weakly increasing. Thus greater sizing precision expands the range of experimentation costs over which impact sizing is strictly optimal.*

2. *For every* κ *satisfying* $K_L(1) < \kappa < K_U(1)$, *where the boundary values at* 1 *are understood as left limits, there exists a unique threshold* $\alpha^{IS}(\kappa;\beta,\gamma_h) \in [\gamma_h, 1)$ *such that impact sizing is strictly optimal if and only if* $\alpha_1 > \alpha^{IS}(\kappa;\beta,\gamma_h)$.

Proposition 5 formalizes the wedge-shaped intermediate-cost region shown in Figure 3. As sizing precision increases, sizing-dependent policies become more valuable relative to sizing-independent alternatives. Consequently, $K_L(\alpha_1)$ weakly decreases and $K_U(\alpha_1)$ weakly increases, expanding the range of experimentation costs over which impact sizing is strictly optimal.

The non-monotonicity in experimentation cost κ reflects the dual role of experimentation. When experimentation is cheap, the firm tests projects directly, so eliciting the manager's manipulable sizing report adds little. When experimentation is too costly, testing no longer disciplines reporting or effort incentives. Impact sizing is valuable only between these extremes, where experimentation is costly enough that the firm wants to economize on it, but still credible enough to support incentive provision.

A natural follow-up question is how precise the sizing signal must be before it can generate any value. The following corollary gives a sharp lower bound.

COROLLARY 3. *For any given (β, γ_h), define the universal precision lower bound by*

$$\underline{\alpha}(\beta, \gamma_h) \equiv \inf_{\kappa:\, K_L(1;\beta,\gamma_h)<\kappa<K_U(1;\beta,\gamma_h)} \alpha^{IS}(\kappa; \beta, \gamma_h),$$

where $\alpha^{IS}(\kappa; \beta, \gamma_h)$ is the precision threshold in Proposition 5. Then $\underline{\alpha}(\beta, \gamma_h) > 1/2$. If $\alpha_1 < \underline{\alpha}(\beta, \gamma_h)$, impact sizing yields no payoff improvement relative to the uninformative-sizing benchmark for any experimentation cost $\kappa > 0$. Moreover,

$$\lim_{\gamma_h \downarrow 1/2} \underline{\alpha}(\beta, \gamma_h) = \frac{1}{2}, \qquad \lim_{\gamma_h \uparrow 1} \underline{\alpha}(\beta, \gamma_h) = 1.$$

Corollary 3 implies that the firm may optimally ignore a marginally informative sizing signal. Even when $\alpha_1 > 1/2$, sizing creates value only if its precision exceeds $\underline{\alpha}(\beta, \gamma_h)$. Otherwise, the informational benefit of using the report is outweighed by the distortions required to elicit it truthfully.

The limiting results sharpen this trade-off. As $\gamma_h \downarrow 1/2$, effort barely improves project quality, making screening more valuable and driving the minimum useful precision toward $1/2$. As $\gamma_h \uparrow 1$, effort nearly ensures high quality, leaving little value from additional screening and driving the threshold toward one.

5.4. Interaction Between Sizing and Effort

The preceding results characterize when the firm optimally uses impact sizing. We next examine how informative sizing changes the firm's decision to induce quality-improving effort. Relative to the uninformative-sizing benchmark, sizing may make effort either more or less attractive. The following definition formalizes these two possibilities using the optimal effort outcomes selected under Assumption 1.

Definition 3 (Complementarity and substitution between sizing and effort). Fix (β, γ_h) and consider (α_1, κ) with $\alpha_1 > 1/2$. Compare the optimal regime at (α_1, κ) with that under the *uninformative-sizing benchmark* $(1/2, \kappa)$, using the effort outcomes selected under Assumption 1.

1. (Complementarity) Sizing complements effort if NI is strictly optimal under the uninformative-sizing benchmark, whereas DI is strictly optimal with informative sizing.

2. (Substitution) Sizing substitutes for effort if EI is strictly optimal under the uninformative-sizing benchmark, whereas SI is strictly optimal with informative sizing.

Figure 4 illustrates the two interactions defined above. Positively sloped hatching marks complementarity, where informative sizing changes the optimal regime from NI to DI. Negatively sloped hatching marks substitution, where informative sizing changes the optimal regime from EI to SI. The following propositions provide sufficient conditions for each effect.

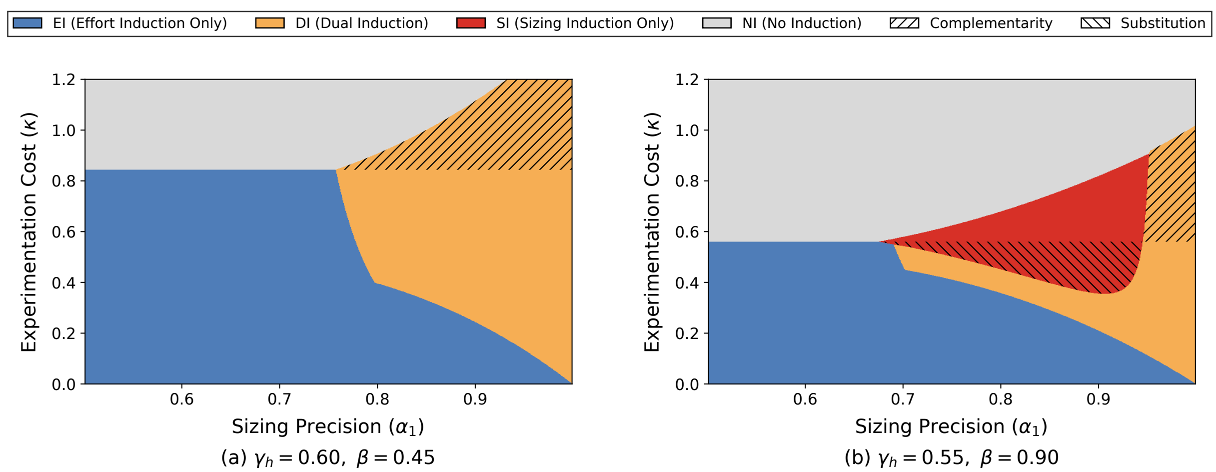


Figure 4 The Interaction between Effort and Sizing

Sizing complements effort. When experimentation is moderately costly, the firm may abandon effort induction if it cannot use sizing. A sufficiently precise sizing signal can restore effort incentives by allowing the firm to target experimentation toward projects that are more likely to be good. Define the uninformative-sizing effort cutoff

$$\kappa^E(\beta, \gamma_h) \equiv \frac{\beta(1-\gamma_h) + 2\gamma_h - 1}{\beta}. \tag{17}$$

This is the experimentation cost above which the uninformative-sizing benchmark no longer induces effort.

PROPOSITION 6 **(Sizing Complements Effort)**. *Under Assumption 1, fix (β, γ_h) with $\beta \in (0,1)$ and $\gamma_h \in (1/2, 1)$. Recall $\kappa^E(\beta, \gamma_h)$ in* (17)*, for every*

$$\kappa \in \left(\kappa^E(\beta, \gamma_h), \frac{\kappa^E(\beta, \gamma_h)}{\gamma_h} \right), \tag{18}$$

there exists a precision threshold $\alpha^C(\kappa; \beta, \gamma_h) \in (1/2, 1)$ such that, for all $\alpha_1 > \alpha^C(\kappa; \beta, \gamma_h)$, sizing complements effort. That is, NI is strictly optimal under the uninformative-sizing benchmark ($\alpha_1 = 1/2$), whereas DI is strictly optimal with sufficiently informative sizing.

Figure 4 illustrates Proposition 6. For experimentation costs in $(\kappa^E, \kappa^E/\gamma_h)$, NI is strictly optimal under the uninformative-sizing benchmark, whereas sufficiently precise sizing makes DI strictly optimal, generating the hatched complementarity region. The strength of the effort incentive varies within (18). It is strict when $\kappa \leq 1/\gamma_h - 1$; when $\kappa > 1/\gamma_h - 1$, it binds and Assumption 1 selects effort. The proof of Proposition 6 derives the corresponding policy constructions.

The mechanism operates through two channels. Sizing allows the firm to target experimentation toward projects with favorable reports, reducing the expected cost of motivating effort. In turn, effort increases the probability of a favorable signal and therefore the value of sizing-based screening. Sizing and effort consequently reinforce each other.

Sizing substitutes for effort. Sizing can also weaken the firm's incentive to induce effort. When effort is costly and the sizing signal is sufficiently informative, the firm may prefer to rely on report-based screening rather than pay the agency cost required to improve project quality.

PROPOSITION 7 **(Sizing Substitutes for Effort)**. *Under Assumption 1, there is a nonempty open region of the parameter space in which sizing substitutes for effort. Specifically, there exist thresholds*

$$\frac{1}{2} < \underline{\beta}_s < \bar{\beta}_s < 1, \quad \frac{1}{2} < \underline{\gamma}_s < \bar{\gamma}_s < 1, \quad \frac{1}{2} < \underline{\alpha}_s < \bar{\alpha}_s < 1, \quad 0 < \underline{\kappa}_s < \bar{\kappa}_s,$$

such that sizing substitutes for effort whenever

$$\beta \in (\underline{\beta}_s, \bar{\beta}_s), \quad \gamma_h \in (\underline{\gamma}_s, \bar{\gamma}_s), \quad \alpha_1 \in (\underline{\alpha}_s, \bar{\alpha}_s), \quad \kappa \in (\underline{\kappa}_s, \bar{\kappa}_s).$$

Within this region, the uninformative-sizing benchmark induces effort, whereas the optimal policy with sizing is SI: the firm uses the sizing signal but does not induce quality-improving effort.

Figure 4(b) illustrates Proposition 7. In the hatched region, EI is optimal under the uninformative-sizing benchmark, whereas informative sizing makes SI optimal. In the construction underlying the proposition, the manager is indifferent between effort and no effort under both policies. The firm prefers effort under the benchmark and no effort under SI, so Assumption 1 selects the corresponding outcomes.

Substitution arises when effort is costly and only modestly improves project quality. Without informative sizing, effort is the firm's only way to improve ex ante quality. Informative sizing provides an alternative: the firm can screen projects using the manager's report and target experimentation toward favorable reports. When screening yields a higher payoff than quality improvement, the firm switches from EI to SI. Proposition 7 establishes a nonempty region in which this occurs, rather than a global threshold characterization.

Together, Propositions 6 and 7 show that sizing changes both the return to and the need for quality-improving effort. It complements effort when better screening raises the value of quality improvement, but substitutes for effort when report-based screening is more attractive than costly quality improvement. Firms should therefore design sizing policies and effort incentives jointly, because improving one instrument may either reinforce or reduce the value of the other.

Comparison with the first best. The complementarity and substitution effects above are second-best incentive effects. In the first-best benchmark of Proposition 3, the decision maker directly chooses quality-improving effort and accounts for its cost. Because $\beta < 1$, effort is always efficient, both under the uninformative-sizing benchmark $\alpha_1 = 1/2$ and under any informative sizing signal $\alpha_1 > 1/2$. Sizing therefore affects only the interim allocation of decisions, namely whether the firm adopts, experiments, or rejects after each signal realization.

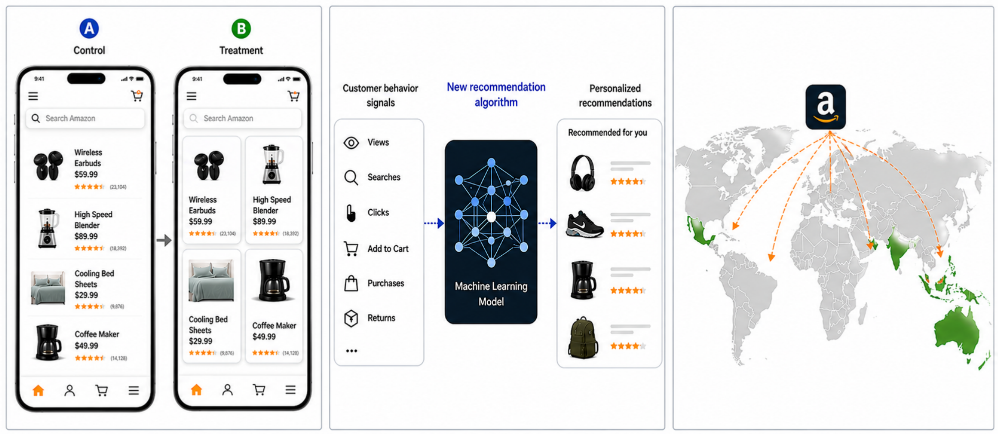
By contrast, in the strategic setting, the firm must use the same implementation and experimentation policy to elicit the manager's private sizing report and to motivate costly effort. As a result, sizing can change the cost and feasibility of effort incentives. This is why sizing can either complement effort, as in Proposition 6, or substitute for effort, as in Proposition 7.

6. Numerical Illustrations

This section illustrates how the theoretical mechanisms map into stylized project classes and how the optimal policy regime changes across plausible parameter environments. We first use benchmark project classes to examine how sizing precision, experimentation cost, and effort cost shape the optimal policy. We then extend the endogenous-quality analysis beyond costless manipulation and examine how positive manipulation costs affect the equilibrium regimes and the value of impact sizing.

6.1. Benchmark Parameterization

To illustrate the model's insights, we consider three stylized project classes that arise in technology firms or multi-unit enterprises. These include digital platforms, online retailers, restaurant chains, hotel groups, logistics networks, and other firms that deploy operational changes through a centralized system across many users, locations, or markets. The three classes differ in implementation cost, experimentation duration, and uncertainty.



(a) List layout test (b) New algorithm design (c) Enter a new geographic market

Figure 5 Examples of experimentation projects within a single firm

- *Interface or process tweaks* are small changes to the way an existing system is presented or executed. Examples include changing button wording on an app, modifying page layout, adjusting default sorting rules, changing pickup instructions in a restaurant app, or revising the display order of products on an e-commerce site. These projects are inexpensive and easy to test.
- *Feature or policy redesigns* change the functionality of the system more substantially. Examples include a new recommendation algorithm, a redesigned checkout flow, a new delivery-fee policy, a revised staffing rule for stores, or a new inventory-replenishment rule across warehouses. These projects require more engineering or operational coordination.
- *Strategic initiatives* involve broad operational or business-model changes. Examples include redesigning an entire platform, entering a new geographic market, launching a new subscription tier, changing the fulfillment model, or rolling out a new store format. These projects are costly to test, difficult to reverse, and highly uncertain in outcome.

The same three project classes arise across operational settings. In online retail, they correspond, respectively, to list-layout tests, recommendation-algorithm redesigns, and geographic expansion, as illustrated in Figure 5. In restaurant operations, the corresponding examples are changes to pickup instructions, redesigns of staffing or menu-recommendation rules, and the introduction of new store formats.

Table 3 reports the parameter values used in the numerical illustrations. Throughout, we set $\gamma_l = 0.50$ and $\alpha_2 = 1$. For each project class, we consider $C = 0$, the costless-manipulation benchmark analyzed above, and $C = 0.20$, a costly-manipulation case. We examine two effort-and-sizing environments. The baseline parameterization sets $\gamma_h = 0.70$ and $\alpha_1 = 0.65$, representing relatively effective effort and moderately informative sizing. The high-precision parameterization, with $\gamma_h = 0.60$ and $\alpha_1 = 0.80$, combines a smaller quality improvement with more precise sizing. We use project scope to discipline the parameterization: within each environment, broader initiatives are harder to test and require more costly preparation, so both experimentation cost and effort cost increase with project scope.

All payoffs are normalized by the value of a successful project. Accordingly, the experimentation cost κ, manipulation cost C, and effort cost $x = \beta(\gamma_h - \gamma_l)$ are cost-to-value ratios rather than direct monetary estimates. The parameter β measures the cost per unit increase in the probability of high quality. Within each parameter configuration, $\gamma_h - \gamma_l$ is fixed, so a larger β implies a larger effort cost x. The manipulation cost C captures the cost of distorting a sizing report, with larger values representing stronger auditing, documentation requirements, or reputational penalties.

Table 3 Parameter Values for Numerical Illustrations

Calibration	Project class	γ_h	α_1	κ	β	C
Baseline	Interface tweak	0.70	0.65	0.05	0.25	$\{0, 0.20\}$
	Feature redesign	0.70	0.65	0.20	0.55	$\{0, 0.20\}$
	Strategic initiative	0.70	0.65	0.40	0.90	$\{0, 0.20\}$
High-precision	Interface tweak	0.60	0.80	0.05	0.25	$\{0, 0.20\}$
	Feature redesign	0.60	0.80	0.45	0.55	$\{0, 0.20\}$
	Strategic initiative	0.60	0.80	0.60	0.90	$\{0, 0.20\}$

Note. These parameter values are chosen to illustrate the economic mechanisms developed in Sections 4 and 5, rather than to provide point estimates for a specific industry.

6.2. Exogenous Project Quality

We first consider the exogenous-quality setting. For interface tweaks, experimentation is cheap in both parameter configurations, so the optimal policy is a sizing-independent, always-experiment policy. The same is true for feature redesigns in the baseline parameterization. In these cases, impact sizing adds little because direct experimentation is inexpensive.

For larger projects, impact sizing becomes valuable when blanket experimentation is too costly, but a sizing-dependent policy remains useful. In the baseline parameterization, strategic initiatives move from sizing-independent blanket experimentation at $C = 0$ to a sizing-dependent selective-experimentation policy at $C = 0.20$. The payoff increases from 0.100 to 0.114, a 14% increase. In the high-sizing-value parameterization, feature redesigns and strategic initiatives are already sizing-dependent under $C = 0$, and costly manipulation further raises their payoffs. The payoff increases from 0.115 to 0.175 for feature redesigns, a 52.2% increase, and from 0.040 to 0.100 for strategic initiatives, a 150.0% increase.

The managerial implication is twofold. First, sizing analytics are most useful for projects whose experimentation costs are high enough to make blanket testing unattractive but not so high that experimentation is abandoned altogether. Second, reporting discipline amplifies the value of sizing: when manipulation is costly, the firm can condition decisions on the manager's signal with fewer incentive distortions.

6.3. Endogenous Project Quality: Costless Manipulation

We next consider endogenous project quality under the analytical benchmark $C = 0$. Under the baseline parameterization, impact sizing is not used. Interface tweaks and feature redesigns are in EI: the firm induces effort and experiments after both reports, so $\tau_1^e(1) = \tau_1^e(-1) = 1$. Strategic initiatives are still in EI, and the firm still induces effort, but uses the minimum experimentation probability needed for incentives, $\tau_1^e(1) = \tau_1^e(-1) = 0.90$. Thus, in the baseline environment, the optimal policy is sizing-independent for all three project classes.

The high-sizing-value parameterization shows when sizing becomes active. Interface tweaks remain in EI because experimentation is cheap. Feature redesigns move to DI: the firm induces effort and conditions its policy on the sizing report, using more experimentation after favorable reports and less after unfavorable reports. Strategic initiatives move to SI: effort is no longer induced, and the firm relies on signal-based screening instead. This illustrates the substitution mechanism: when effort is costly and the sizing signal is precise, the firm may rely on screening rather than inducing quality-improving effort.

6.4. Endogenous Project Quality with Costly Manipulation: An Illustrative Extension

Finally, we examine how the endogenous-quality results change when manipulation is costly. We solve the firm's optimization problem numerically at $C = 0.20$, without imposing truthful reporting. The numerical procedure considers both effort choices and all four deterministic reporting strategies and applies Assumption 1 among the manager's best responses. For comparability, the $C = 0$ and $C = 0.20$ analyses use the same policy space and differ only in the manipulation cost. In the high-sizing-value parameterization, interface tweaks remain EI. Feature redesigns remain DI, with payoff increasing from 0.177 to 0.222, a 25.4% increase. Strategic initiatives remain SI, with payoff increasing from 0.036 to 0.091, which corresponds to a 153.3% relative increase.

Overall, making manipulation more costly can improve firm payoff by reducing the incentive distortions required to use managerial information. As project scope increases, the firm relies less on blanket experimentation and more on report-contingent combinations of direct adoption and selective experimentation. Thus, stronger reporting discipline makes impact sizing more valuable by allowing the firm to use the manager's signal more effectively.

7. Concluding Remarks

We study how a firm should design its sizing-based and experiment-based policies when a manager privately observes a sizing signal, is biased toward implementation, and may exert costly effort to improve project quality. Three findings emerge. First, even when report manipulation is costless, impact sizing can strictly improve the firm's payoff, but only within a wedge-shaped region of intermediate experimentation costs. Greater sizing precision expands this region, while positive manipulation costs make truthful reporting easier to sustain. Second, when manipulation costs alone cannot sustain the first best, truthful reporting may require over-experimentation after favorable reports or partial adoption after unfavorable reports. Experimentation therefore both reveals project quality and disciplines reporting. Third, when project quality is endogenous, experimentation also provides effort incentives. Sizing can complement effort by raising the return to quality improvement or substitute for effort by making report-based screening more attractive than costly quality improvement.

These findings suggest that firms should use impact sizing selectively and govern how sizing reports affect subsequent decisions. Sizing is most valuable when testing every proposal is too costly but selective experimentation remains credible enough to discipline reporting. Firms can facilitate truthful reporting by raising the effective cost of manipulation or by adjusting the experimentation and implementation consequences of favorable and unfavorable reports. Firms should therefore coordinate sizing governance, experimentation rules, and effort incentives, because more precise sizing can either strengthen effort incentives or replace effort with report-based screening.

Our analysis considers a single project, takes sizing precision as exogenous, and assumes that the firm can commit to its policy. Future work could study competition among managers for scarce experimentation capacity, limited commitment to report-contingent decisions, and endogenous investment in sizing precision. These extensions would further connect impact sizing to resource allocation, organizational governance, and analytics investment.

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Appendix

This appendix provides the technical results supporting the main analysis. Appendix A.1 proves the exogenous-quality results in Section 4. Appendix A.2 proves the endogenous-quality results in Section 5. Appendix A.3 establishes the normalization of monotone experiment-based policies, including when experimentation is imperfect.

Appendix A: Technical Details

A.1. Proof of Section 4 (Exogenous Quality)

We prove the results in Section 4 sequentially. The proofs of Corollaries 1 and 2 use the regional policies and payoffs derived below.

A.1.1. Proof of Proposition 1 Since $\gamma_l = 1/2$, (4) gives $\Gamma(1;\gamma_l) = \alpha_1$ and $\Gamma(-1;\gamma_l) = 1-\alpha_1$. Consider first a positive sizing signal. Direct adoption yields payoff $2\alpha_1 - 1$, while rejection yields zero. If the firm experiments, the FB experiment-based policy is to adopt after a positive experimental outcome and reject after a negative one, i.e., $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. The experimentation payoff is therefore $\alpha_1 - \kappa$. Comparing experimentation with direct adoption, the firm experiments after $s_1 = 1$ if and only if $\alpha_1 - \kappa > 2\alpha_1 - 1$, or equivalently $\kappa < 1 - \alpha_1$. Otherwise, direct adoption is optimal.

Now consider a negative sizing signal. Direct adoption yields payoff $1 - 2\alpha_1 < 0$, so rejection dominates direct adoption. Experimentation yields payoff $1 - \alpha_1 - \kappa$. Hence the firm experiments after $s_1 = -1$ if and only if $\kappa < 1 - \alpha_1$; otherwise, rejection is optimal. The experiment-based policy $\tau_2(1) = 1$ and $\tau_2(-1) = 0$ is optimal because the experiment perfectly reveals project quality.

Finally, since $\Pr(s_1 = 1 \mid \gamma_l) = \Pr(s_1 = -1 \mid \gamma_l) = 1/2$, the ex-ante payoff is $(2\alpha_1 - 1)/2$ when $\kappa \geq 1 - \alpha_1$, and $1/2 - \kappa$ when $\kappa < 1 - \alpha_1$. This proves the result.

A.1.2. Proof of Lemma 1. Fix the reporting strategy σ selected under Assumption 1 for $\mathcal{T} = (\tau_1, \tau_2)$. For each true signal $s \in S_1$, define

$$\tilde{\tau}_1(s) = \sum_{r \in S_1} \sigma(r \mid s)\tau_1(r),$$

and set $\tilde{\mathcal{T}} = (\tilde{\tau}_1, \tau_2)$. Each $\tilde{\tau}_1(s)$ is a convex combination of feasible allocations and is therefore feasible. Truthful reporting under $\tilde{\mathcal{T}}$ induces the same signal-contingent allocation as σ under $\mathcal{T}$.

Let $G_s(r)$ be the manager's gross implementation payoff when the true signal is s and the original report is r. For $s = 1$, both reports are costless. The lottery $\sigma(\cdot \mid 1)$ therefore uses only reports that maximize $G_1(r)$. Truthful reporting under $\tilde{\mathcal{T}}$ attains this maximum, whereas a deviation produces a convex combination of the same two original-report payoffs. Hence type 1 weakly prefers truth telling.

For $s = -1$, let $M_{-1} = \max_{r \in S_1}\{G_{-1}(r) - c(r \mid -1)\}$. The lottery $\sigma(\cdot \mid -1)$ uses only reports attaining M_{-1}. Removing the manipulation cost from this lottery can only increase the manager's payoff, so truthful reporting under $\tilde{\mathcal{T}}$ gives at least M_{-1}. A deviation to report 1 produces a convex combination of $G_{-1}(-1)$ and $G_{-1}(1)$, followed by cost C. Since $G_{-1}(-1) \leq M_{-1}$ and $G_{-1}(1) \leq M_{-1} + C$, the deviation gives at most M_{-1}. Type -1 also weakly prefers truth telling.

It remains to verify selection when a reporting constraint binds. A direct report corresponds to a lottery $\sigma(\cdot \mid s')$ over reports available under the original policy. For type 1, the two representations have the same reporting cost. For type -1, the expected cost of the original-report lottery corresponding to a direct report of 1 is weakly below C. Thus, if a direct deviation tied with truth telling and gave the firm a higher payoff, the corresponding original-report lottery would also be a best response and would give the firm the same higher payoff. This contradicts the selection of σ under Assumption 1. Truthful reporting is therefore selected. Because the firm receives the same allocation after each true signal, its payoff is unchanged. This proves the lemma.

A.1.3. Proof of Proposition 2 We prove the proposition in two steps. First, we characterize the experiment-based component of an Ex policy. We then solve for its sizing-based component.

Step 1 (Experiment-based component of the Ex policy). Write $a_r = \tau_1^a(r)$, $e_r = \tau_1^e(r)$, and $d_s = \tau_2(s)$ for $r,s \in \{-1,1\}$. We first show that there exists an Ex policy satisfying $\tau_2(1) = 1$ and $\tau_2(-1) = 0$.

Consider any feasible policy. Suppose first that $d_1 \geq d_{-1}$. For each report r, set $\widetilde{a}_r = a_r + d_{-1}e_r$ and $\widetilde{e}_r = (d_1 - d_{-1})e_r$, and use the experiment-based rule $\widetilde{\tau}_2(1) = 1$ and $\widetilde{\tau}_2(-1) = 0$. The transformed policy is feasible because $\widetilde{a}_r + \widetilde{e}_r = a_r + d_1 e_r \leq a_r + e_r \leq 1$. The transformation preserves the final implementation probability conditional on each report and each project quality. It therefore preserves the manager's reporting incentives and the firm's implementation payoff. Because it weakly reduces the probability of formal experimentation, it weakly increases the firm's payoff.

Now suppose that $d_1 < d_{-1}$. Moving the common implementation probability d_1 into direct adoption similarly reduces the policy to the anti-monotone rule $\tau_2(1) = 0$ and $\tau_2(-1) = 1$, without changing implementation probabilities and while weakly reducing experimentation costs. We show that this rule is dominated. Under the anti-monotone rule, experimentation after report 1 can be replaced by direct adoption with probability $(1 - \alpha_1)e_1$. This preserves the positive-signal manager's implementation payoff from report 1, reduces the negative-signal manager's payoff from deviating to report 1, and strictly increases the firm's payoff. We may therefore set $e_1 = 0$.

The truthful-reporting constraints then require $(1 - \alpha_1)e_{-1} \leq a_1 - a_{-1} \leq C + \alpha_1 e_{-1}$, and the firm's payoff is $[(2\alpha_1 - 1)(a_1 - a_{-1}) - (\alpha_1 + \kappa)e_{-1}]/2$. If $a_1 - a_{-1} \leq C$, eliminating e_{-1} preserves feasibility and weakly increases the payoff. If $a_1 - a_{-1} > C$, the upper reporting constraint gives

$$\Pi \leq \frac{1}{2}(2\alpha_1 - 1)C - \frac{1}{2}\big(2\alpha_1(1 - \alpha_1) + \kappa\big)e_{-1} \leq \frac{1}{2}(2\alpha_1 - 1)C. \tag{A.1}$$

The last bound is attained by the no-experiment policy $(a_1, e_1, a_{-1}, e_{-1}) = (C, 0, 0, 0)$. Hence the anti-monotone rule is weakly dominated by a policy that does not experiment. Because the experiment-based rule is irrelevant when no experiment occurs, this policy can also be represented using $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. Thus, an Ex policy can be chosen with $\tau_2(1) = 1$ and $\tau_2(-1) = 0$.

Step 2 (Sizing-based component of the Ex policy). Substituting the experiment-based rule from Step 1 into Problem (OP-Ex), and using $\gamma_l = 1/2$, gives

$$\begin{aligned}
\max_{a_1, e_1, a_{-1}, e_{-1}} \quad & \frac{1}{2}\Big[a_1(2\alpha_1 - 1) + e_1(\alpha_1 - \kappa) + a_{-1}(1 - 2\alpha_1) + e_{-1}(1 - \alpha_1 - \kappa)\Big] \\
\text{s.t.} \quad & a_{-1} + \alpha_1 e_{-1} \leq a_1 + \alpha_1 e_1, \\
& a_1 + (1 - \alpha_1)e_1 - a_{-1} - (1 - \alpha_1)e_{-1} \leq C, \\
& a_1 + e_1 \leq 1, \qquad a_{-1} + e_{-1} \leq 1, \\
& a_1, e_1, a_{-1}, e_{-1} \geq 0.
\end{aligned} \tag{OP-Ex-LP}$$

The first two constraints ensure truthful reporting after positive and negative sizing signals, respectively.

If $\kappa \leq 1 - \alpha_1$, experimentation is weakly optimal after either sizing signal. The policy $(a_1, e_1, a_{-1}, e_{-1}) = (0, 1, 0, 1)$ is feasible and attains the first-best payoff. It is therefore an Ex policy, giving Region I.

Now suppose that $\kappa > 1 - \alpha_1$. We first show that the negative-signal truthful-reporting constraint must bind at every optimal solution. Suppose, to the contrary, that it is slack at an optimal solution. Under this supposition, we must have $a_1 + e_1 = 1$. Otherwise, the firm could increase a_1 by an amount smaller than both the remaining probability slack and the slack in the negative-signal reporting constraint. This change would relax the positive-signal reporting constraint and strictly increase the firm's payoff because $2\alpha_1 - 1 > 0$, contradicting optimality.

Still under the supposition of slackness, we must also have $e_1 = 0$. If $e_1 > 0$, the firm could shift a small probability $\varepsilon > 0$ from experimentation to direct adoption after a positive report. Specifically, it could increase a_1 by ε and decrease e_1 by ε. This keeps $a_1 + e_1$ unchanged and relaxes the positive-signal reporting constraint. It increases the left-hand side of the negative-signal constraint by only $\alpha_1\varepsilon$, so that constraint remains satisfied for sufficiently small ε. The firm's payoff increases by $\varepsilon[\kappa - (1 - \alpha_1)]/2 > 0$, again contradicting optimality. Hence slackness would require $a_1 = 1$ and $e_1 = 0$.

The slack negative-signal constraint would then imply $a_{-1} + (1 - \alpha_1)e_{-1} > 1 - C$, so at least one of a_{-1} and e_{-1} must be positive. Slightly reducing a positive a_{-1} or e_{-1} would relax the positive-signal reporting constraint and preserve the negative-signal constraint for a sufficiently small reduction. It would also strictly increase the firm's payoff because $1 - 2\alpha_1 < 0$ and $1 - \alpha_1 - \kappa < 0$. This gives the final contradiction.

Therefore, when $\kappa > 1 - \alpha_1$, the negative-signal truthful-reporting constraint binds: $a_1 + (1 - \alpha_1)e_1 - a_{-1} - (1 - \alpha_1)e_{-1} = C$. Substituting this equality into the objective gives

$$\Pi = \frac{1}{2}\Big[C(2\alpha_1 - 1) + \big(1 - 2\alpha_1 + 2\alpha_1^2 - \kappa\big)e_1 + \big(2\alpha_1(1 - \alpha_1) - \kappa\big)e_{-1}\Big].$$

Thus, once truthful reporting by the negative-signal manager is imposed, the firm's remaining payoff depends on the sizing-based policy only through e_1 and e_{-1}.

The binding reporting constraint also gives $a_1 + e_1 = a_{-1} + C + \alpha_1 e_1 + (1 - \alpha_1)e_{-1}$. Because $a_1 + e_1 \leq 1$ and $a_{-1} \geq 0$, every feasible policy must satisfy $\alpha_1 e_1 + (1 - \alpha_1)e_{-1} \leq 1 - C$. We first maximize the reduced objective over $e_1, e_{-1} \in [0, 1]$ subject to this necessary condition. This provides an upper bound on the original problem. After solving this reduced problem, we show that each solution can be completed with feasible adoption probabilities, so the upper bound is attained.

We begin by comparing experimentation after the two reports. Increasing e_1 by ε tightens the feasibility bound by $\alpha_1\varepsilon$, whereas increasing e_{-1} by ε tightens it by $(1 - \alpha_1)\varepsilon$. When both experiment probabilities have positive objective coefficients, their relative contributions are therefore determined by

$$\frac{1 - 2\alpha_1 + 2\alpha_1^2 - \kappa}{\alpha_1} - \frac{2\alpha_1(1 - \alpha_1) - \kappa}{1 - \alpha_1} = \frac{(2\alpha_1 - 1)\big[\kappa - (1 - \alpha_1)\big]}{\alpha_1(1 - \alpha_1)} > 0.$$

Hence, for $\kappa > 1 - \alpha_1$, the firm assigns experimentation probability to a positive report before assigning it to a negative report.

We now solve the reduced problem by considering the signs of the two objective coefficients.

Case 1: $1-\alpha_1 < \kappa \le 2\alpha_1(1-\alpha_1)$. Both experiment probabilities have nonnegative objective coefficients, and experimentation after a positive report has the larger relative contribution. The firm therefore increases e_1 as much as possible before increasing e_{-1}.

If $C \le 1-\alpha_1$, the feasibility bound permits $e_1 = 1$. The remaining slack then permits $e_{-1} = 1 - C/(1-\alpha_1)$. Thus the reduced solution is $(e_1, e_{-1}) = (1, 1 - C/(1-\alpha_1))$.

If $C > 1-\alpha_1$, the bound does not permit $e_1 = 1$. All available slack is assigned to e_1, giving $(e_1, e_{-1}) = ((1-C)/\alpha_1, 0)$.

Case 2: $2\alpha_1(1-\alpha_1) < \kappa \le 1 - 2\alpha_1 + 2\alpha_1^2$. The coefficient on e_{-1} is negative, while the coefficient on e_1 is nonnegative. Hence the firm sets $e_{-1} = 0$ and makes e_1 as large as the feasibility bound permits.

If $C \le 1-\alpha_1$, this gives $(e_1, e_{-1}) = (1, 0)$. If $C > 1-\alpha_1$, it gives $(e_1, e_{-1}) = ((1-C)/\alpha_1, 0)$.

Case 3: $\kappa > 1 - 2\alpha_1 + 2\alpha_1^2$. Both experiment probabilities have negative objective coefficients. The reduced problem therefore sets $(e_1, e_{-1}) = (0, 0)$.

It remains to show that these reduced solutions are feasible in the original problem. Each solution identified above satisfies $e_1 \ge e_{-1}$. For any such solution, select $a_1 = 1 - e_1$ and $a_{-1} = 1 - C - \alpha_1 e_1 - (1-\alpha_1)e_{-1}$. The reduced feasibility bound ensures $a_{-1} \ge 0$. Moreover, $a_{-1} + e_{-1} = 1 - C - \alpha_1(e_1 - e_{-1}) \le 1$. The negative-signal reporting constraint binds by construction, while the positive-signal reporting constraint holds because $a_1 + \alpha_1 e_1 - a_{-1} - \alpha_1 e_{-1} = C + (2\alpha_1 - 1)(e_1 - e_{-1}) \ge 0$. Thus every reduced solution can be completed to a feasible policy and attains the upper bound.

Applying this construction gives the four policies in Table 2. In Case 1 with $C \le 1-\alpha_1$, it gives $(a_1, e_1, a_{-1}, e_{-1}) = (0, 1, 0, 1 - C/(1-\alpha_1))$, which is Region II. In Case 2 with $C \le 1-\alpha_1$, it gives $(0, 1, 1-\alpha_1-C, 0)$, which is Region III. In Cases 1 and 2 with $C > 1-\alpha_1$, it gives $(1-(1-C)/\alpha_1, (1-C)/\alpha_1, 0, 0)$, which is Region IV. Finally, Case 3 gives $(1, 0, 1-C, 0)$, which is Region V.

In Region V, $(C, 0, 0, 0)$ is also optimal. When there is no experimentation, the binding negative-signal constraint requires $a_1 - a_{-1} = C$, and the firm's payoff depends only on this difference. Therefore $(1, 0, 1-C, 0)$ and $(C, 0, 0, 0)$ yield the same payoff. The former is a convenient optimal representative with $a_1 + e_1 = 1$, but this probability constraint need not bind at every optimum.

For reference, the corresponding regional payoffs are

$$
\begin{aligned}
\Pi_1 &= \frac{1}{2} - \kappa + \frac{C(\alpha_1 + \kappa - 1)}{2(1-\alpha_1)}, \\
\Pi_2 &= \alpha_1(\alpha_1 + C - 1) - \frac{1}{2}(C + \kappa - 1), \\
\Pi_3 &= \frac{(C-1)(\kappa-1)}{2\alpha_1} + \alpha_1 + \frac{C}{2} - 1, \\
\Pi_4 &= \left(\alpha_1 - \frac{1}{2}\right) C.
\end{aligned}
\tag{A.2}
$$

This establishes the five regions in Table 2 and completes the proof. $\square$

A.1.4. Proof of Corollary 1 We prove the four claims in order.

First, if $\kappa \le 1-\alpha_1$, the FB and Ex policies both experiment after either sizing signal. If $\kappa > 1-\alpha_1$, the FB policy does not experiment, whereas every experimentation probability under the Ex policy is nonnegative.

This proves weak over-experimentation. In the intermediate-cost range $1-\alpha_1 < \kappa \le 1-2\alpha_1+2\alpha_1^2$, the Ex policy experiments after a positive report with probability one in Regions II and III and with probability $(1-C)/\alpha_1$ in Region IV. This probability is strictly positive when $C<1$.

Second, suppose $C=1$. In Region I, the Ex policy coincides with the FB full-experimentation policy. For $\kappa > 1-\alpha_1$, the Region IV and V policies both reduce to direct adoption after a positive report and rejection after a negative report. This is a first-best allocation. Hence the Ex policy attains the FB payoff for every κ.

Third, the FB policy admits a no-experimentation optimizer when $\kappa \ge 1-\alpha_1$. Consider the Ex problem with $C<1$. The best no-experimentation candidate is P_4, with payoff $\Pi_4 = C(\alpha_1 - 1/2)$. If $\kappa \le 1-\alpha_1$, full experimentation strictly dominates P_4 because

$$\frac{1}{2} - \kappa - \Pi_4 \ge (1-C)\left(\alpha_1 - \frac{1}{2}\right) > 0.$$

If $1-\alpha_1 < \kappa < 1-2\alpha_1+2\alpha_1^2$ and $C \le 1-\alpha_1$, then $\Pi_2 - \Pi_4 = (1-2\alpha_1+2\alpha_1^2-\kappa)/2 > 0$, and P_1 weakly dominates P_2 whenever Region II applies. If instead $C > 1-\alpha_1$, then

$$\Pi_3 - \Pi_4 = \frac{(1-C)(1-2\alpha_1+2\alpha_1^2-\kappa)}{2\alpha_1} > 0.$$

Thus, no policy without experimentation is optimal below $1-2\alpha_1+2\alpha_1^2$. At $\kappa = 1-2\alpha_1+2\alpha_1^2$, P_4 ties P_2 or P_3, and for larger κ, P_4 is optimal. Finally,

$$\frac{d(1-\alpha_1)}{d\alpha_1} = -1, \qquad \frac{d(1-2\alpha_1+2\alpha_1^2)}{d\alpha_1} = 4\alpha_1 - 2 > 0,$$

which proves the divergent cutoff comparison.

It remains to prove the agency-cost result. The agency cost is zero in Region I. Outside Region I, the FB payoff is $\alpha_1 - 1/2$. Differentiating its difference from the regional Ex payoffs in (A.2) gives

$$\frac{\partial(\Pi^{\mathrm{FB}} - \Pi^{\mathrm{Ex}})}{\partial \alpha_1} = \begin{cases} 1 - \dfrac{C\kappa}{2(1-\alpha_1)^2}, & \text{Region II,} \\ 2 - 2\alpha_1 - C, & \text{Region III,} \\ \dfrac{(1-C)(1-\kappa)}{2\alpha_1^2}, & \text{Region IV,} \\ 1 - C, & \text{Region V.} \end{cases}$$

In Region II, $C \le 1-\alpha_1$ and $\kappa \le 2\alpha_1(1-\alpha_1)$, so the first derivative is at least $1-\alpha_1 > 0$. In Region III, $C \le 1-\alpha_1$, so the second derivative is also at least $1-\alpha_1 > 0$. In Region IV, $\kappa \le 1-2\alpha_1+2\alpha_1^2 < 1$, so the third derivative is nonnegative. The Region V derivative is nonnegative because $C \le 1$. The derivatives need not agree at the regional boundaries. However, direct substitution into the payoff formulas in (A.2) shows that the adjacent Ex payoffs coincide at every policy-switching boundary. The agency cost is therefore continuous in α_1. Because it is piecewise differentiable and has a nonnegative derivative in the interior of every region, it is weakly increasing in α_1. When $C=1$, the regional policies coincide with the first-best allocation, so the agency cost is zero. This completes the proof. □

A.1.5. Proof of Corollary 2 For the first claim, the firm can ignore the sizing report. Full experimentation after both reports yields payoff $1/2 - \kappa$, and rejection without experimentation yields zero. Both policies are feasible in the Ex problem. Therefore

$$\Pi^{\mathrm{Ex}} \geq \max\{1/2 - \kappa, 0\}.$$

We next establish the stated equality cases. Suppose first that $C = 0$. If $\kappa \leq 2\alpha_1(1-\alpha_1)$, Regions I and II both reduce to full experimentation, so $\Pi^{\mathrm{Ex}} = 1/2 - \kappa$. Since $2\alpha_1(1-\alpha_1) \leq 1/2$, this equals the uninformative-sizing benchmark payoff. If $\kappa \geq 1 - 2\alpha_1 + 2\alpha_1^2$, policy P_4 is optimal and yields zero. At equality, it ties the experimenting candidate, so an optimal no-experimentation policy still exists. Since $1 - 2\alpha_1 + 2\alpha_1^2 \geq 1/2$, the uninformative-sizing benchmark payoff is also zero. Now suppose $C > 0$ and $\kappa \leq 1 - \alpha_1$. Region I applies, so the Ex policy experiments after both reports and obtains $1/2 - \kappa$. Because $1 - \alpha_1 < 1/2$, this again equals the uninformative-sizing benchmark payoff.

Finally, direct differentiation of the regional payoffs in (A.2) shows that they are weakly increasing in α_1 and C, and weakly decreasing in κ. The only non-immediate signs occur in Regions II and IV. In Region II, $\alpha_1 + \kappa - 1 > 0$ and $C \leq 1 - \alpha_1$; in Region IV, $C > 1 - \alpha_1$, $\kappa > 1 - \alpha_1$, and hence $(1-C)(1-\kappa) < \alpha_1^2$. These inequalities give the stated signs. The payoff formulas agree at every regional boundary by the comparisons in the proof of Proposition 2, so the same monotonicity holds for the global Ex payoff. $\square$

A.2. Proof of Section 5

A.2.1. Proof of Lemma 2 Fix the original policy $\mathcal{T} = (\tau_1, \tau_2)$, the selected effort-induced prior γ^*, and the selected reporting strategy σ. For each true sizing signal $s \in S_1$, define

$$\tilde{\tau}_1(s) = \sum_{r \in S_1} \sigma(r \mid s)\tau_1(r),$$

and set $\tilde{\mathcal{T}} = (\tilde{\tau}_1, \tau_2)$. Since each $\tau_1(r)$ is a feasible probability vector over adoption, experimentation, and rejection, $\tilde{\tau}_1(s)$ is feasible.

We first verify truthful reporting under the intended effort γ^*. Because $C = 0$, reporting costs are zero and the manager's reporting payoff is linear in the induced allocation. Under the original policy, every report used with positive probability by type s under $\sigma(\cdot \mid s)$ is a best response. Therefore the lottery $\tilde{\tau}_1(s)$ gives type s the same payoff as the equilibrium reporting lottery under $\mathcal{T}$. Any deviation under $\tilde{\mathcal{T}}$ induces $\tilde{\tau}_1(s')$ for some $s' \in S_1$, which is a convex combination of allocations available under the original policy. Hence the payoff from such a deviation cannot exceed the maximum reporting payoff under $\mathcal{T}$. Truthful reporting is therefore a best response for each signal s.

We next show that the effort choice is preserved. Under prior γ^*, truthful reporting under $\tilde{\mathcal{T}}$ induces the same signal-contingent allocation as σ induces under $\mathcal{T}$. Thus the manager's expected payoff from γ^* is unchanged. Consider the alternative effort-induced prior $\gamma' \neq \gamma^*$. For each signal, every report under $\tilde{\mathcal{T}}$ induces a convex combination of allocations available under $\mathcal{T}$. Since the manager's payoff is linear in the allocation, the best reporting payoff under $\tilde{\mathcal{T}}$ is weakly below the best reporting payoff under $\mathcal{T}$. Therefore the manager's payoff from the alternative effort choice is weakly lower under $\tilde{\mathcal{T}}$ than under $\mathcal{T}$. Since γ^* was optimal under $\mathcal{T}$, it remains optimal under $\tilde{\mathcal{T}}$.

It remains to verify principal-preferred selection when an effort or reporting constraint binds. Consider any complete effort and reporting strategy under the direct policy. Replace every direct report t by the original-report lottery $\sigma(\cdot \mid t)$. Because $C = 0$, this composed strategy gives both the manager and the firm exactly the same payoffs under the original policy. Therefore, if a strategy tied with $(\gamma^*, \text{truthful reporting})$ under the direct policy and gave the firm a higher payoff, its composed strategy would tie with (γ^*, σ) and give the firm the same higher payoff under the original policy. This contradicts Assumption 1. Hence the intended effort and truthful-reporting strategy are selected under $\tilde{\mathcal{T}}$.

Conditional on γ^* and signal s, truthful reporting under $\tilde{\mathcal{T}}$ induces the same reduced-form allocation as $\sigma(\cdot \mid s)$ under $\mathcal{T}$. Linearity of the firm's payoff then gives the same ex-ante payoff. This proves the lemma. $\square$

A.2.2. Proof of Proposition 3

Experiment-based policy. Because $\alpha_2 = 1$, the formal experiment perfectly reveals project quality. Following a positive experimental outcome, adoption yields payoff one and rejection yields zero. Following a negative outcome, adoption yields payoff minus one and rejection yields zero. Hence the optimal experiment-based policy is $\tau_2(1) = 1$ and $\tau_2(-1) = 0$.

At any posterior belief Γ, the decision maker therefore compares direct-adoption payoff $2\Gamma - 1$, rejection payoff zero, and experimentation payoff $\Gamma - \kappa$.

Efficiency of quality-improving effort. Under no effort, $\gamma_l = 1/2$, so $\Gamma(1;\gamma_l) = \alpha_1$ and $\Gamma(-1;\gamma_l) = 1 - \alpha_1$. After a positive sizing signal, the gain from experimentation relative to direct adoption is $1 - \alpha_1 - \kappa$. After a negative sizing signal, the gain from experimentation relative to rejection is also $1 - \alpha_1 - \kappa$. It follows that, when $\kappa < 1 - \alpha_1$, the optimal policy experiments after both signals. When $\kappa > 1 - \alpha_1$, it adopts directly after a positive signal and rejects after a negative signal. At $\kappa = 1 - \alpha_1$, both policies are optimal.

Fix one such optimal policy at prior γ_l, and denote it by $\mathcal{T}_l$. If $\mathcal{T}_l$ experiments after both signals, its expected project payoff under any prior γ is $\gamma - \kappa$. If it adopts after a positive signal and rejects after a negative signal, its expected payoff is $\gamma + \alpha_1 - 1$. Therefore, in either case, $\Pi(\mathcal{T}_l;\gamma_h) - \Pi(\mathcal{T}_l;\gamma_l) = \gamma_h - \gamma_l$.

Applying the same policy under the high-quality prior gives

$$\begin{aligned}\max_{\mathcal{T}\in\mathcal{F}}\left[\Pi(\mathcal{T};\gamma_h) - x\right] &\geq \Pi(\mathcal{T}_l;\gamma_h) - \beta(\gamma_h - \gamma_l) \\ &= \max_{\mathcal{T}\in\mathcal{F}}\Pi(\mathcal{T};\gamma_l) + (1-\beta)(\gamma_h - \gamma_l) > \max_{\mathcal{T}\in\mathcal{F}}\Pi(\mathcal{T};\gamma_l).\end{aligned}$$

The final inequality follows from $\beta < 1$ and $\gamma_h > \gamma_l$. Thus effort is strictly optimal, and the prior is γ_h.

Sizing-based policy after effort. Consider first a positive sizing signal. Because $\Gamma(1;\gamma_h) > 1/2$, direct adoption dominates rejection. Experimentation is preferred to direct adoption when $\kappa < 1 - \Gamma(1;\gamma_h)$. Hence the lower cutoff is $\underline{\kappa} = 1 - \Gamma(1;\gamma_h)$.

Now consider a negative sizing signal. Bayes' rule implies that $\Gamma(-1;\gamma_h) \geq 1/2$ if and only if $\gamma_h \geq \alpha_1$. If $\gamma_h \geq \alpha_1$, direct adoption weakly dominates rejection, and experimentation is preferred to direct adoption when $\kappa < 1 - \Gamma(-1;\gamma_h)$. Hence $\overline{\kappa} = 1 - \Gamma(-1;\gamma_h)$.

If $\gamma_h < \alpha_1$, then $\Gamma(-1;\gamma_h) < 1/2$, so rejection dominates direct adoption. Experimentation is preferred to rejection when $\kappa < \Gamma(-1;\gamma_h)$. Hence $\overline{\kappa} = \Gamma(-1;\gamma_h)$.

When $\gamma_h = \alpha_1$, the negative-signal posterior equals $1/2$, so direct adoption and rejection yield the same payoff. We select direct adoption as stated in the proposition.

Ordering of the cutoffs. It remains to verify that $\underline{\kappa} \leq \overline{\kappa}$. If $\gamma_h \geq \alpha_1$, a positive signal produces a higher posterior than a negative signal. Therefore, $1 - \Gamma(1;\gamma_h) \leq 1 - \Gamma(-1;\gamma_h)$.

If $\gamma_h < \alpha_1$, bringing $\Gamma(1;\gamma_h) + \Gamma(-1;\gamma_h) - 1$ to a common denominator gives the nonnegative numerator $\alpha_1(1-\alpha_1)(2\gamma_h - 1)$. Hence $1 - \Gamma(1;\gamma_h) \leq \Gamma(-1;\gamma_h)$.

Combining the two cutoff comparisons gives the three policy regions stated in the proposition. At either cutoff, the adjacent actions yield the same payoff, so multiple first-best policies may be optimal. □

A.2.3. Proof of Proposition 4 The proof reduces the firm's problem to a finite comparison of candidate policies. Because strict sizing dependence makes the DI and SI feasible sets nonclosed, we solve the corresponding closed relaxations and classify their optimizers ex post. Assumption 1 determines the selected effort and reporting strategies whenever the manager is indifferent.

The proof proceeds in three steps. First, we establish a canonical experiment-based rule under which the firm adopts after a positive experimental outcome and rejects after a negative outcome. Second, we solve the four regime problems. For DI and SI, the incentive and feasibility constraints reduce the closed relaxations to two-dimensional feasible sets. Their relevant vertices yield three DI candidates and four SI candidates. We solve the sizing-independent EI and NI problems separately. Finally, we compare all feasible and correctly classified candidates, including the NI regime value. The positive branch of the NI value is dominated by EI–2, while its zero branch corresponds to rejection. The resulting finite upper envelope gives the optimal payoff stated in Proposition 4.

Step 1: Canonical experiment-based policy. We first normalize the experiment-based component of the policy. This reduction will be used in solving both the DI and SI problems.

LEMMA A.1 **(Canonical Experiment-Based Policy)**. *Consider the closed relaxation of either* (OP-DI) *or* (OP-SI) *under $C = 0$ and $\alpha_2 = 1$. There exists an optimal policy whose experiment-based component satisfies $\tau_2(1) = 1$ and $\tau_2(-1) = 0$.*

Proof. Write $d_1 = \tau_2(1)$ and $d_{-1} = \tau_2(-1)$. We consider separately whether the experiment-based rule is monotone or anti-monotone in its outcome.

Monotone orientation. Suppose first that $d_1 \geq d_{-1}$. For each report r, set $\widetilde{a}_r = a_r + d_{-1}e_r$ and $\widetilde{e}_r = (d_1 - d_{-1})e_r$, and normalize the experiment-based rule to $\widetilde{\tau}_2(1) = 1$ and $\widetilde{\tau}_2(-1) = 0$. The transformed policy is feasible because $\widetilde{a}_r + \widetilde{e}_r = a_r + d_1 e_r \leq a_r + e_r \leq 1$.

Conditional on report r, the original policy implements a high-quality project with probability $a_r + d_1 e_r$ and a low-quality project with probability $a_r + d_{-1}e_r$. The transformed policy produces exactly the same two implementation probabilities. It therefore preserves the manager's payoff from every effort and reporting strategy, as well as the firm's implementation payoff.

The probability of formal experimentation falls from e_r to $(d_1 - d_{-1})e_r$, which is weakly smaller because $0 \leq d_1 - d_{-1} \leq 1$. The transformed policy therefore weakly increases the firm's payoff. Applying this transformation to an optimizer establishes the desired canonical rule whenever $d_1 \geq d_{-1}$.

Anti-monotone orientation. Now suppose that $d_1 < d_{-1}$. For each report r, move the common implementation probability $d_1 e_r$ into direct adoption by setting $\widehat{a}_r = a_r + d_1 e_r$ and $\widehat{e}_r = (d_{-1} - d_1)e_r$, and normalize the experiment-based rule to $\widehat{\tau}_2(1) = 0$ and $\widehat{\tau}_2(-1) = 1$. This policy is feasible because $\widehat{a}_r + \widehat{e}_r = a_r + d_{-1}e_r \leq a_r + e_r \leq 1$. As above, it preserves implementation probabilities conditional on project quality and weakly reduces experimentation costs. It remains to show that this normalized anti-monotone rule cannot improve upon the canonical rule.

The relaxed DI problem. Under the anti-monotone rule, experimentation leads to implementation only when the project is low quality. Thus, after observing sizing signal s and submitting report r, the manager's implementation probability is $\widehat{a}_r + [1 - \Gamma(s;\gamma)]\widehat{e}_r$.

For brevity, let $\Delta a = \widehat{a}_1 - \widehat{a}_{-1}$ and $\Delta e = \widehat{e}_{-1} - \widehat{e}_1$. Truthful reporting under prior γ_h requires $\left(1 - \Gamma_1^h\right)\Delta e \leq \Delta a \leq \left(1 - \Gamma_{-1}^h\right)\Delta e$. Because $\Gamma_1^h > \Gamma_{-1}^h$, these inequalities also imply $\Delta e \geq 0$.

Under truthful reporting, the manager's expected implementation probability at prior γ is

$$V(\gamma) = \left[1 - \alpha_1 + (2\alpha_1 - 1)\gamma\right]\widehat{a}_1 + \left[\alpha_1 - (2\alpha_1 - 1)\gamma\right]\widehat{a}_{-1} + (1-\gamma)\left[(1-\alpha_1)\widehat{e}_1 + \alpha_1\widehat{e}_{-1}\right].$$

This function is affine in γ. Using $\Delta a \leq (1 - \Gamma_{-1}^h)\Delta e \leq \Delta e$, its slope satisfies

$$\begin{aligned} V'(\gamma) &= (2\alpha_1 - 1)\Delta a - (1-\alpha_1)\widehat{e}_1 - \alpha_1\widehat{e}_{-1} \leq (2\alpha_1 - 1)\Delta e - (1-\alpha_1)\widehat{e}_1 - \alpha_1\widehat{e}_{-1} \\ &= -\alpha_1\widehat{e}_1 - (1-\alpha_1)\widehat{e}_{-1} \leq 0. \end{aligned}$$

Hence $V(\gamma_h) \leq V(\gamma_l)$.

If the manager exerts effort and reports truthfully, his payoff is $V(\gamma_h) - x$. Without effort, he can always use the truthful reporting rule and obtain $V(\gamma_l)$, even if another reporting rule would give a higher payoff. Therefore, $U_M^E = V(\gamma_h) - x < V(\gamma_l) \leq U_M^{NE}$, where the strict inequality follows from $x > 0$. Thus the anti-monotone rule cannot satisfy the effort incentive constraint. It is therefore infeasible for the relaxed DI problem.

The relaxed SI problem. Under SI, the relevant prior is $\gamma_l = 1/2$. Truthful reporting under the anti-monotone rule implies $\Delta e \geq 0$ and $\Delta a \leq \alpha_1\Delta e$. The firm's expected payoff is therefore bounded above by

$$\begin{aligned} \Pi &= \frac{1}{2}\Big[(2\alpha_1 - 1)\Delta a - (1-\alpha_1)\widehat{e}_1 - \alpha_1\widehat{e}_{-1} - \kappa(\widehat{e}_1 + \widehat{e}_{-1})\Big] \\ &\leq -\frac{1}{2}\Big[(2\alpha_1^2 - 2\alpha_1 + 1)\widehat{e}_1 + 2\alpha_1(1-\alpha_1)\widehat{e}_{-1} + \kappa(\widehat{e}_1 + \widehat{e}_{-1})\Big] \leq 0. \end{aligned}$$

The last inequality follows because $2\alpha_1^2 - 2\alpha_1 + 1 = \alpha_1^2 + (1-\alpha_1)^2 > 0$ and all experiment probabilities and costs are nonnegative.

Rejecting every project attains payoff zero and can be represented using $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. Hence an optimum of the relaxed SI problem can also be chosen with the canonical experiment-based rule.

Combining the monotone and anti-monotone cases proves the lemma. □

Step 2: Solution of four regime problems

Candidates from the relaxed DI problem. We solve the closed relaxation of (OP-DI) under $C = 0$. For $s \in \{-1, 1\}$, let $p_s(\gamma) = \Pr(s_1 = s \mid \gamma)$, where $p_1(\gamma) = \gamma\alpha_1 + (1-\gamma)(1-\alpha_1)$ and $p_{-1}(\gamma) = \gamma(1-\alpha_1) + (1-\gamma)\alpha_1$. To shorten the expressions below, write $\Gamma_s^j = \Gamma(s;\gamma_j)$ and $p_s^j = p_s(\gamma_j)$ for $j \in \{l, h\}$. As above, let $a_r = \tau_1^a(r)$ and $e_r = \tau_1^e(r)$ for $r \in \{-1, 1\}$.

The relaxed DI problem. By Lemma A.1, we may restrict attention to the experiment-based policy $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. The manager's implementation probability after observing sizing signal s and reporting r is then $m_j(s,r) = a_r + \Gamma^j_s e_r$ under prior γ_j.

The closed DI relaxation is

$$\begin{aligned}
\max_{\{a_s,e_s\}_{s\in\{-1,1\}}} \quad & \sum_{s\in\{-1,1\}} p^h_s \left[a_s(2\Gamma^h_s - 1) + e_s(\Gamma^h_s - \kappa) \right] \\
\text{s.t.} \quad & a_1 + \Gamma^h_1 e_1 \geq a_{-1} + \Gamma^h_1 e_{-1}, \\
& a_{-1} + \Gamma^h_{-1} e_{-1} \geq a_1 + \Gamma^h_{-1} e_1, \\
& \sum_{s\in\{-1,1\}} p^h_s \left(a_s + \Gamma^h_s e_s \right) - x \geq \sum_{s\in\{-1,1\}} p^l_s \left(a_{\rho(s)} + \Gamma^l_s e_{\rho(s)} \right), \qquad \forall \rho : \{-1,1\} \to \{-1,1\}, \\
& a_s + e_s \leq 1, \qquad a_s, e_s \geq 0, \qquad s \in \{-1,1\}.
\end{aligned} \tag{OP-DI-LP}$$

The first two constraints ensure truthful reporting under effort. The third set ensures that effort is preferred to no effort followed by any reporting rule. Because the report space is binary, the low-effort manager has four deterministic reporting rules: truthful reporting, pooling on 1, pooling on -1, and swapping reports. Since $\gamma_l = 1/2$, the corresponding effort constraints are

$$\begin{aligned}
(E^T): \quad & \sum_s p^h_s m_h(s,s) - x \geq \frac{1}{2} m_l(1,1) + \frac{1}{2} m_l(-1,-1), \\
(E^P): \quad & \sum_s p^h_s m_h(s,s) - x \geq \frac{1}{2} m_l(1,1) + \frac{1}{2} m_l(-1,1), \\
(E^N): \quad & \sum_s p^h_s m_h(s,s) - x \geq \frac{1}{2} m_l(1,-1) + \frac{1}{2} m_l(-1,-1), \\
(E^S): \quad & \sum_s p^h_s m_h(s,s) - x \geq \frac{1}{2} m_l(1,-1) + \frac{1}{2} m_l(-1,1).
\end{aligned}$$

The feasible set is a compact polytope, so an optimum exists at one of its vertices. We next derive two binding restrictions that reduce the four-dimensional problem.

The positive-report feasibility constraint. For a sizing-dependent truthful policy, let $d = e_1 - e_{-1}$ and $R = a_{-1} - a_1$. The two truthful-reporting constraints require $\Gamma^h_{-1} d \leq R \leq \Gamma^h_1 d$. They also imply $d > 0$, because $d = 0$ would require $R = 0$, making the policy sizing-independent.

Increasing a_1 and a_{-1} by the same amount leaves all reporting and effort comparisons unchanged and raises the firm's expected payoff at rate $2\gamma_h - 1 > 0$. Moreover, $(a_1 + e_1) - (a_{-1} + e_{-1}) = d - R \geq (1 - \Gamma^h_1)d \geq 0$. Thus the positive-report probability constraint has weakly less slack than the negative-report probability constraint. The two constraints may bind simultaneously, but every sizing-dependent DI optimum must satisfy $a_1 + e_1 = 1$.

Eliminating the low-precision region. Under truthful reporting, the manager's expected implementation probability is affine in the quality prior. Because the effort cost is $x = \beta(\gamma_h - \gamma_l)$, constraint (E^T) is equivalent to

$$(2\alpha_1 - 1)(a_1 - a_{-1}) + \alpha_1 e_1 + (1 - \alpha_1) e_{-1} \geq \beta. \tag{A.3}$$

The left-hand side is the difference between the manager's implementation probabilities conditional on high and low project quality. It is therefore at most one.

Consider the sizing-independent policy whose experimentation probability equals the left-hand side of (A.3) and whose direct-adoption probability equals one minus that amount. This policy is feasible because its experimentation probability lies between β and one, and it induces effort.

Using $a_1 + e_1 = 1$, direct substitution shows that the payoff difference between this EI policy and the current DI policy is $(2\gamma_h - 1)(1-\alpha_1)(d-R) + \kappa(2\alpha_1 - 1)\left[R - (1-\gamma_h)d\right]$. Suppose $\alpha_1 \leq \gamma_h$. Then $\Gamma^h_{-1} \geq 1/2$, so $R \geq \Gamma^h_{-1} d \geq d/2 > (1-\gamma_h)d$. In addition, $R \leq \Gamma^h_1 d < d$. Hence the payoff difference is strictly positive. Every sizing-dependent DI policy is therefore strictly dominated by a sizing-independent EI policy. Thus only $\alpha_1 > \gamma_h$ can generate a relevant sizing-dependent DI candidate.

The negative-signal reporting constraint. Suppose henceforth that $\alpha_1 > \gamma_h$, so $\Gamma^h_{-1} < 1/2$. Using $a_1 - a_{-1} = -R$ and $e_{-1} = e_1 - d$, the left-hand side of (A.3) can be written as $e_1 - (1-\alpha_1)d - (2\alpha_1 - 1)R$, which is at most e_1. It follows from (A.3) that $e_1 \geq \beta$.

We now show that the negative-signal truthful-reporting constraint binds. Suppose instead that $R - \Gamma^h_{-1} d > 0$. Reduce a_{-1} by exactly this amount. The new direct-adoption probability is $a_1 + \Gamma^h_{-1} d \geq 0$, and negative-signal truth telling becomes binding. Positive-signal truth telling and negative-report feasibility are relaxed.

It remains to verify the effort constraints. Before the change, the slack in (E^P) is $p^h_{-1}[R - \Gamma^h_{-1}d] + (\gamma_h - 1/2)(e_1 - \beta)$. The reduction in a_{-1} decreases this slack by $p^h_{-1}[R - \Gamma^h_{-1}d]$, leaving $(\gamma_h - 1/2)(e_1 - \beta) \geq 0$.

The slacks in (E^T) and (E^S) increase by $(1/2 - p^h_{-1})[R - \Gamma^h_{-1}d]$, while the slack in (E^N) increases by $(1 - p^h_{-1})[R - \Gamma^h_{-1}d]$. These changes are nonnegative because $p^h_{-1} = \alpha_1 - (2\alpha_1 - 1)\gamma_h < 1/2$. The perturbation therefore preserves all four effort constraints.

The coefficient on a_{-1} in the firm's objective is $p^h_{-1}(2\Gamma^h_{-1} - 1) = \gamma_h - \alpha_1 < 0$. Therefore, reducing a_{-1} increases the firm's payoff by $(\alpha_1 - \gamma_h)[R - \Gamma^h_{-1}d] > 0$, contradicting optimality. Hence the negative-signal reporting constraint binds: $a_1 + \Gamma^h_{-1}e_1 = a_{-1} + \Gamma^h_{-1}e_{-1}$.

The reduced feasible set. Combining the two binding constraints gives $a_1 = 1 - e_1$ and $a_{-1} = 1 - e_1 + \Gamma^h_{-1}(e_1 - e_{-1})$.

Because $\alpha_1 > \gamma_h > 1/2$, we have $1 - \alpha_1 < \Gamma^h_{-1} < \alpha_1$. For a sizing-dependent policy, the negative-signal truth-telling equality under γ_h therefore lies strictly between the two truth-telling bounds under γ_l. Truthful reporting is consequently the low-effort manager's unique best response. At a sizing-independent boundary, all reporting rules induce the same allocation. Thus, in either case, (E^T) is the only distinct effort constraint.

The exact reduced feasible set is

$$0 \leq e_{-1} \leq e_1 \leq 1, \qquad \alpha_1 e_1 + (1-\alpha_1)e_{-1} - (2\alpha_1 - 1)\Gamma^h_{-1}(e_1 - e_{-1}) \geq \beta. \tag{A.4}$$

The first inequalities ensure feasible allocation probabilities, and the final inequality is (E^T).

Vertices of the reduced feasible set. The reduced polygon has four relevant boundaries: $e_{-1} = 0$, $e_1 = 1$, $e_{-1} = e_1$, and a binding effort constraint. Along $e_{-1} = e_1$, the two endpoints are $e_1 = e_{-1} = 1$ and $e_1 = e_{-1} = \beta$, corresponding to EI–1 and EI–2.

The remaining vertices are obtained from the three pairwise intersections of $e_1 = 1$, $e_{-1} = 0$, and the binding effort constraint. These intersections give the following sizing-dependent candidates:

$$\text{DI-1}: \quad (a_1, e_1, a_{-1}, e_{-1}) = \left(0, 1, \frac{\Gamma^h_{-1}(1-\beta)}{1-\alpha_1+(2\alpha_1-1)\Gamma^h_{-1}}, \frac{\beta-\alpha_1+(2\alpha_1-1)\Gamma^h_{-1}}{1-\alpha_1+(2\alpha_1-1)\Gamma^h_{-1}}\right). \tag{A.5}$$

$$\text{DI-2}: \quad (a_1, e_1, a_{-1}, e_{-1}) = \left(0, 1, \Gamma^h_{-1}, 0\right). \tag{A.6}$$

$$\text{DI-3}: \quad (a_1, e_1, a_{-1}, e_{-1}) = \left(1 - \frac{\beta}{\alpha_1-(2\alpha_1-1)\Gamma^h_{-1}}, \frac{\beta}{\alpha_1-(2\alpha_1-1)\Gamma^h_{-1}}, 1 - \frac{(1-\Gamma^h_{-1})\beta}{\alpha_1-(2\alpha_1-1)\Gamma^h_{-1}}, 0\right). \tag{A.7}$$

The denominators are positive because $1-\alpha_1 < \Gamma^h_{-1} < 1/2$. These formulas are algebraically equivalent to their expanded expressions in $(\alpha_1, \gamma_h, \beta)$.

All three DI candidates satisfy the positive-report feasibility constraint and the negative-signal truthful-reporting constraint at equality. DI–1 additionally has $e_1 = 1$ and (E^T) binding. DI–2 has $e_1 = 1$ and $e_{-1} = 0$; its effort constraint is binding only when $\beta = \alpha_1 - (2\alpha_1 - 1)\Gamma^h_{-1}$. DI–3 has $e_{-1} = 0$ and (E^T) binding.

At parameter boundaries, additional constraints may bind and adjacent candidates may coincide, but no new policy formula arises. Each candidate is retained only when its coordinates satisfy (A.4). If an effort or reporting constraint binds, Assumption 1 determines whether the candidate is classified as DI or assigned to another regime.

For $j \in \{1,2,3\}$, let Π^{DI}_j denote the firm's payoff from DI–j, evaluated under prior γ_h using (12). We summarize the result below.

PROPOSITION A.1 **(Candidates from the Relaxed DI Problem)**. *Suppose $C = 0$. The closed relaxation of the DI problem admits an optimal experiment-based policy satisfying $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. Its relevant extreme-point candidates are EI–1, EI–2, and DI–1 through DI–3. Each candidate is retained in the final comparison only if it is feasible and Assumption 1 assigns its selected effort and reporting strategies to the stated regime.*

Effort Induction Only (EI). We solve (OP-EI). Since the policy is sizing-independent, write $a \equiv \tau^a_1(1) = \tau^a_1(-1)$ and $e \equiv \tau^e_1(1) = \tau^e_1(-1)$. By Lemma A.1, we restrict attention without loss to $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. The manager's expected implementation payoff under prior γ is therefore $a + e\gamma$. Hence the effort constraint (EC) is $a + e\gamma_h - x \geq a + e\gamma_l$. Using $x = \beta(\gamma_h - \gamma_l)$, this reduces to $e \geq \beta$.

The firm's payoff under effort is $a(2\gamma_h - 1) + e(\gamma_h - \kappa)$. Since $2\gamma_h - 1 > 0$, feasibility implies $a = 1 - e$. Thus the EI problem reduces to

$$\max_{e \in [\beta,1]} \{2\gamma_h - 1 + e(1 - \gamma_h - \kappa)\}.$$

Therefore, if $\kappa < 1 - \gamma_h$, the optimal policy sets $e = 1$ and $a = 0$; if $\kappa > 1 - \gamma_h$, it sets $e = \beta$ and $a = 1 - \beta$. The two exposed candidates are

$$\text{EI-1}: \ (a, e, a, e) = (0, 1, 0, 1), \qquad \text{EI-2}: \ (a, e, a, e) = (1-\beta, \beta, 1-\beta, \beta).$$

At $\kappa = 1 - \gamma_h$, any $e \in [\beta, 1]$ with $a = 1 - e$ is optimal.

For completeness, the closed EI feasible set has one additional vertex, EI-0 : $(a, e, a, e) = (0, \beta, 0, \beta)$. Its active constraints are $a = 0$ and $e = \beta$. At this boundary the manager is indifferent between effort and no effort, but the firm prefers effort by $\beta(\gamma_h - 1/2) > 0$, so it is classified as EI. It is strictly dominated by EI–2, which adds direct adoption without changing either the experimentation probability or the manager's effort incentive. Hence the two exposed EI vertices are EI–1, with active constraints $a = 0$ and $a + e = 1$, and EI–2, with active constraints $e = \beta$ and $a + e = 1$.

Substituting the two EI policies into (12) gives

$$\Pi_1^{EI} = \gamma_h - \kappa, \qquad \Pi_2^{EI} = 2\gamma_h - 1 + \beta(1 - \gamma_h - \kappa). \tag{A.8}$$

The resulting optimal policy is summarized below.

PROPOSITION A.2 **(Optimal EI Policy)**. *Under EI, the optimal sizing-independent policy is as follows:*

$$(\tau_1^a, \tau_1^e) = \begin{cases} (0, 1), & \kappa < 1 - \gamma_h, \\ (1 - \beta, \beta), & \kappa > 1 - \gamma_h. \end{cases}$$

At $\kappa = 1 - \gamma_h$, any $(\tau_1^a, \tau_1^e) = (1 - e, e)$ with $e \in [\beta, 1]$ is optimal. The EI payoff is given by (A.8). *Moreover, EI weakly dominates NI if and only if*

$$\kappa \leq \frac{2 - \beta}{\beta}\gamma_h + \frac{\beta - 1}{\beta}.$$

Candidates from the relaxed SI problem. We solve the closed relaxation of (OP-SI) under $C = 0$. By Lemma A.1, we may restrict attention to $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. We continue to use $m_j(s, r) = a_r + \Gamma_s^j e_r$ for the manager's implementation probability after observing signal s and reporting r under prior γ_j.

Let T, P, N, S denote truthful reporting, pooling on the positive report, pooling on the negative report, and swapping reports, respectively. Let N^ρ denote the no-effort constraint against effort followed by reporting rule $\rho \in \{T, P, N, S\}$.

The relaxed SI problem. The firm's problem is

$$\begin{aligned} \max_{\{a_s, e_s\}_{s \in \{-1,1\}}} \quad & \sum_{s \in \{-1,1\}} p_s^l \left[a_s(2\Gamma_s^l - 1) + e_s(\Gamma_s^l - \kappa) \right] \\ \text{s.t.} \quad & m_l(1, 1) \geq m_l(1, -1), \\ & m_l(-1, -1) \geq m_l(-1, 1), \\ & \sum_{s \in \{-1,1\}} p_s^l m_l(s, s) \geq \sum_{s \in \{-1,1\}} p_s^h m_h(s, \rho(s)) - x, \ \forall \rho \in \{T, P, N, S\}, \\ & a_s + e_s \leq 1, \qquad a_s, e_s \geq 0, \qquad s \in \{-1, 1\}. \end{aligned} \tag{OP-SI-LP}$$

The first two constraints ensure truthful reporting under no effort. The next four constraints ensure that no effort is preferred to effort followed by any reporting rule.

Normalization of direct adoption. A sizing-dependent policy that induces truthful reporting under $\gamma_l = 1/2$ must satisfy $e_1 > e_{-1}$ and $a_1 < a_{-1}$. Subtracting a_1 from both direct-adoption probabilities preserves every reporting and effort comparison. It also leaves the firm's payoff under no effort unchanged because a common direct-adoption probability has expected payoff $2\gamma_l - 1 = 0$. Therefore, an SI optimum can be represented with $a_1 = 0$.

This is a normalization along a payoff-flat direction. It does not mean that $a_1 = 0$ must hold at every optimal policy before normalization.

The no-effort constraint against pooling on the positive report. We next show that every correctly classified, sizing-dependent SI candidate that can strictly improve upon the sizing-independent envelope has a representative at which N^P binds.

Starting from a normalized policy, consider reducing a_{-1}. This raises the firm's payoff at rate $(2\alpha_1 - 1)/2 > 0$. It relaxes positive-signal truth telling, negative-report feasibility, N^N, and N^S. Only negative-signal truth telling, N^T, and N^P can tighten. We show that neither negative-signal truth telling nor N^T can prevent the reduction at a relevant SI optimum before N^P binds.

Suppose first that N^T binds. Let $d = e_1 - e_{-1} > 0$. Because $a_1 = 0$, the manager's indifference between effort and no effort requires $e_{-1} + \alpha_1 d - (2\alpha_1 - 1)a_{-1} = \beta$. At this effort tie, the firm's no-effort payoff satisfies $2\Pi_l^T = \beta - \kappa(d + 2e_{-1})$.

Under principal-preferred selection, no effort can be selected at this tie only if the firm weakly prefers the no-effort outcome. This requires $a_{-1} + e_{-1} + \alpha_1 d - \kappa(2\alpha_1 - 1)d \leq 0$. The following identity shows the implication of these two conditions:

$$\begin{aligned}
&\big(a_{-1} + e_{-1} + \alpha_1 d\big)(d + 2e_{-1}) - (2\alpha_1 - 1)d\big[e_{-1} + \alpha_1 d - (2\alpha_1 - 1)a_{-1}\big] \\
&\quad = 2\Big[\alpha_1(1 - \alpha_1)d^2 + de_{-1} + e_{-1}^2 + \{\alpha_1^2 + (1 - \alpha_1)^2\}da_{-1} + e_{-1}a_{-1}\Big] > 0.
\end{aligned}$$

The principal-preferred condition and the binding manager constraint therefore imply $\kappa(d + 2e_{-1}) > \beta$, and hence $\Pi_l^T < 0$. If the principal-preferred condition fails, the policy is assigned to effort. If it holds, the policy is strictly dominated by rejection. Thus a policy at which N^T prevents the reduction cannot be a relevant SI candidate.

Now suppose that negative-signal truth telling binds before N^P. Then $a_{-1} = (1 - \alpha_1)d$. Because $\Gamma_{-1}^h > 1 - \alpha_1$, both high-effort signal types strictly prefer report 1, so pooling on the positive report is the high-effort manager's unique best reporting rule. On this face, the slack in N^P is $(\gamma_h - 1/2)(\beta - e_1)$. Thus, if N^P is slack, then $e_1 < \beta$.

The firm's payoff on this face satisfies $2\Pi_l^T = \{\alpha_1^2 + (1 - \alpha_1)^2 - \kappa\}e_1 + \{2\alpha_1(1 - \alpha_1) - \kappa\}e_{-1}$. The first coefficient exceeds the second by $(2\alpha_1 - 1)^2 > 0$. Therefore, if the payoff is positive, the coefficient on e_1 must be positive.

Increase e_1 and simultaneously increase a_{-1} by $1 - \alpha_1$ times the same amount. This preserves binding negative-signal truth telling and strictly increases the firm's payoff. Pooling on the positive report remains the high-effort manager's unique best reporting rule, so N^P is the only relevant no-effort constraint. The adjustment can continue until $e_1 = \beta$, where N^P binds. Neither probability constraint binds earlier because $e_{-1} \leq e_1 < \beta < 1$. Therefore, a positive-payoff policy at which negative-signal truth telling prevents the reduction cannot be optimal while N^P remains slack.

We conclude that every relevant normalized SI candidate can be chosen with N^P binding.

The reduced feasible set. Combining $a_1 = 0$ with binding N^P gives the following exact reduction:

$$\begin{aligned}
&a_{-1} = (2\gamma_h - \alpha_1)e_1 - (1 - \alpha_1)e_{-1} - 2\beta(\gamma_h - 1/2), \\
&0 \leq e_{-1} \leq e_1 \leq 1, \qquad (1 - \alpha_1)(e_1 - e_{-1}) \leq a_{-1} \leq \min\{\alpha_1, \Gamma_{-1}^h\}(e_1 - e_{-1}).
\end{aligned} \tag{A.9}$$

The lower and first upper bounds on a_{-1} are the negative- and positive-signal truthful-reporting constraints under no effort. To understand the second upper bound, observe that a positive-signal manager strictly prefers report 1 under effort. Hence only truthful reporting and pooling on 1 can be optimal under effort. Because N^P binds, N^T is satisfied if and only if $a_{-1} \le \Gamma^h_{-1}(e_1 - e_{-1})$. The other two no-effort constraints are therefore redundant.

The negative-report probability constraint is also redundant. Indeed, the upper report bound implies $a_{-1} \le e_1 - e_{-1}$, and hence $a_{-1} + e_{-1} \le e_1 \le 1$. Thus (A.9) is the exact reduced feasible set for the relevant normalized SI candidates.

When $e_1 = e_{-1}$, the report bounds require $a_{-1} = 0$, and binding N^P then gives $e_1 = e_{-1} = \beta$. This is the sizing-independent boundary $(0, \beta, 0, \beta)$, which is assigned to effort and strictly dominated by EI–2.

Reporting ties. Before enumerating the remaining vertices, we record how principal-preferred selection resolves a reporting tie. If two reports give the manager the same implementation probability $m = a + \Gamma e$ at posterior Γ, the firm's conditional payoff can be written as

$$(2\Gamma - 1)m + \big[2\Gamma(1 - \Gamma) - \kappa\big]e. \tag{A.10}$$

Thus the firm prefers the tied report with the larger experimentation probability when $\kappa < 2\Gamma(1 - \Gamma)$, and the report with the smaller experimentation probability when the inequality is reversed. At equality, the secondary rule selects truthful reporting.

Eliminating the positive-signal truth-telling face. Suppose the upper bound in (A.9) is generated by positive-signal truth telling, so $a_{-1} = \alpha_1(e_1 - e_{-1})$. The positive-signal manager is then indifferent between reports. Because $e_1 > e_{-1}$, the tie-breaking rule in (A.10) selects truthful reporting only when $\kappa \le 2\alpha_1(1 - \alpha_1)$.

On this face, writing $d = e_1 - e_{-1}$, the firm's payoff is

$$\Pi^T_l = \frac{1}{2}\Big[\{2\alpha_1(1 - \alpha_1) - \kappa\}d + (1 - 2\kappa)e_{-1}\Big].$$

Because $d + e_{-1} = e_1 \le 1$ and $2\alpha_1(1 - \alpha_1) - \kappa \ge 0$, this payoff is bounded above by one-half of the larger of $2\alpha_1(1 - \alpha_1) - \kappa$ and $1 - 2\kappa$.

The EI–1 payoff $\gamma_h - \kappa$ strictly exceeds both resulting bounds. It exceeds $1/2 - \kappa$ by $\gamma_h - 1/2 > 0$. It also exceeds $\alpha_1(1 - \alpha_1) - \kappa/2$, because $\kappa \le 2\alpha_1(1 - \alpha_1)$ implies that the difference is at least $\gamma_h - 2\alpha_1(1 - \alpha_1) > 0$. Therefore, every vertex on this face is either excluded by principal-preferred reporting selection or strictly dominated by EI–1.

Remaining vertices. After eliminating the positive-signal truth-telling face, the remaining sizing-dependent vertices are generated by the following pairs of boundaries:

$$\{T^-_l, e_{-1} = 0\}, \qquad \{N^T, e_{-1} = 0\}, \qquad \{N^T, e_1 = 1\}, \qquad \{e_1 = 1, e_{-1} = 0\}.$$

Here T^-_l denotes binding negative-signal truth telling under no effort. The lower and upper report bounds can meet only when $e_1 = e_{-1}$, which gives the sizing-independent boundary identified above. The lower report bound cannot meet $e_1 = 1$, because binding T^-_l and N^P together imply $e_1 = \beta < 1$. Hence the four displayed pairs exhaust the remaining sizing-dependent intersections.

They generate the following candidates:

$$\text{SI-1}: \quad (a_1, e_1, a_{-1}, e_{-1}) = \big(0, \beta, \beta(1-\alpha_1), 0\big). \tag{A.11}$$

$$\text{SI-2}: \quad (a_1, e_1, a_{-1}, e_{-1}) = \left(0, \frac{\beta(\alpha_1 + \gamma_h - 2\alpha_1\gamma_h)}{\alpha_1^2 - 2\alpha_1\gamma_h + \gamma_h}, \frac{\beta\gamma_h(1-\alpha_1)}{\alpha_1^2 - 2\alpha_1\gamma_h + \gamma_h}, 0\right). \tag{A.12}$$

$$\text{SI-3}: \quad (a_1, e_1, a_{-1}, e_{-1}) = \left(0, 1, \frac{\Gamma^h_{-1}(1-\beta)}{1-\alpha_1 + (2\alpha_1 - 1)\Gamma^h_{-1}}, \frac{\beta - \alpha_1 + (2\alpha_1 - 1)\Gamma^h_{-1}}{1-\alpha_1 + (2\alpha_1 - 1)\Gamma^h_{-1}}\right). \tag{A.13}$$

$$\text{SI-4}: \quad (a_1, e_1, a_{-1}, e_{-1}) = (0, 1, -\alpha_1 + \beta + 2\gamma_h - 2\beta\gamma_h, 0). \tag{A.14}$$

SI–1 is generated by binding T_l^-, N^P, and $e_{-1} = 0$. SI–2 is generated by binding N^T, N^P, and $e_{-1} = 0$. SI–3 is generated by binding N^T, N^P, and $e_1 = 1$. SI–4 is generated by binding N^P, $e_1 = 1$, and $e_{-1} = 0$. All four candidates use the normalized representative $a_1 = 0$.

Before normalization, SI–1 and SI–2 lie on payoff-flat edges obtained by adding a common direct-adoption probability. Choosing the representative with $a_1 = 0$ does not change the SI payoff and weakly favors no effort under principal-preferred selection.

At parameter boundaries, additional constraints may bind and adjacent candidates may coincide, but no new policy formula arises. The rejection policy yields zero and is already included through NI, so it need not be repeated in the SI candidate list.

For $j \in \{1,2,3,4\}$, let Π_j^{SI} denote the firm's payoff from SI–j, evaluated under prior γ_l using (12).

Principal-preferred classification. For a candidate policy $v = (a_1, e_1, a_{-1}, e_{-1})$, define the firm's payoff under prior γ_j and reporting rule ρ by

$$\widehat{\Pi}_j^\rho(v;\kappa) = \sum_{s\in\{-1,1\}} p_s^j \left[a_{\rho(s)}(2\Gamma_s^j - 1) + e_{\rho(s)}(\Gamma_s^j - \kappa)\right], \qquad j \in \{l, h\}. \tag{A.15}$$

At a DI candidate where an effort constraint binds, effort is selected if and only if

$$\widehat{\Pi}_h^T(v;\kappa) > \max_{\rho:\, E^\rho \text{ binds at } v} \widehat{\Pi}_l^\rho(v;\kappa). \tag{A.16}$$

The inequality is strict because the secondary rule selects no effort if the firm also remains indifferent.

At an SI candidate, no effort is selected if and only if

$$\widehat{\Pi}_l^T(v;\kappa) \geq \max_{\rho:\, N^\rho \text{ binds at } v} \widehat{\Pi}_h^\rho(v;\kappa). \tag{A.17}$$

The inequality is weak because the secondary rule selects no effort at a remaining effort tie.

For each DI candidate, negative-signal truth telling binds. Since the positive report has the larger experimentation probability, the reporting-tie identity selects the truthful negative report if and only if $\kappa \geq 2\Gamma^h_{-1}(1 - \Gamma^h_{-1})$. Whenever an effort constraint also binds, the candidate is assigned to DI only when (A.16) holds for every binding deviation.

For SI–1, both negative-signal truth telling and N^P bind. Combining the reporting and effort comparisons shows that SI–1 is assigned to SI only when $\kappa \geq 2\alpha_1(1-\alpha_1) + (2\gamma_h - 1)$.

For SI–2 through SI–4, no effort is selected precisely when (A.17) holds for every binding no-effort constraint. Any reporting tie is resolved using (A.10). For generic SI–4 parameters, only N^P binds, and the firm's payoff advantage from selecting no effort is

$$\widehat{\Pi}_l^T - \widehat{\Pi}_h^P = \frac{2\alpha_1^2 + 4\alpha_1\beta\gamma_h - 2\alpha_1\beta - 4\alpha_1\gamma_h - 2\beta\gamma_h + \beta + \kappa}{2}.$$

These are classification conditions, not additional optimality assumptions. If a condition fails, Assumption 1 selects a different effort or reporting strategy that gives the firm a weakly higher payoff. The same policy is then feasible in the relaxation corresponding to that selected strategy, whose optimum is already included in the global candidate comparison. Ex-post reclassification therefore cannot reduce the global upper envelope.

The resulting candidate set is summarized below.

PROPOSITION A.3 **(Candidates from the Relaxed SI Problem)**. *Suppose $C = 0$. The closed relaxation of the SI problem admits an optimal experiment-based policy satisfying $\tau_2(1) = 1$ and $\tau_2(-1) = 0$. Its relevant sizing-dependent candidates are SI–1 through SI–4. The sizing-independent boundary candidate is dominated by EI–2, and the zero-payoff rejection policy is accounted for through NI. Each SI candidate is retained in the final comparison only if it is feasible and Assumption 1 assigns its selected effort and reporting strategies to SI.*

No Induction (NI). Under NI, the firm neither induces quality-improving effort nor uses the sizing signal. Let a and e denote its direct-adoption and experimentation probabilities. The manager's implementation payoff under prior γ is $a + e\gamma$. Hence no effort is strictly preferred when $e < \beta$. At $e = \beta$, the manager is indifferent. Under the same policy, however, the firm's payoff from effort exceeds its payoff from no effort by $(2a + e)(\gamma_h - \gamma_l) > 0$. Assumption 1 therefore assigns the boundary $e = \beta$ to EI rather than NI.

The closed NI relaxation is defined by $a, e \geq 0$, $a + e \leq 1$, and $e \leq \beta$. Its four vertices are $(0,0)$, $(1,0)$, $(0,\beta)$, and $(1-\beta,\beta)$. The first two correspond to rejection and unconditional adoption, both of which yield zero payoff under $\gamma_l = 1/2$. At the latter two vertices, the effort constraint binds. Principal-preferred selection classifies them as EI–0 and EI–2, respectively, and EI–0 is dominated by EI–2.

Under no effort, direct adoption has zero expected payoff because $\gamma_l = 1/2$, whereas experimentation yields $1/2 - \kappa$. The NI regime value is therefore

$$\Pi^{NI} = \sup_{0 \leq e < \beta} e(1/2 - \kappa) = \beta \max\{1/2 - \kappa, 0\}.$$

When $\kappa < 1/2$, this value is approached as $e \uparrow \beta$ but is not attained within NI. When $\kappa \geq 1/2$, it equals zero and is attained by rejecting the project. For $\kappa < 1/2$, the positive NI supremum is strictly dominated by EI–2 because $\Pi_2^{EI} - \beta(1/2 - \kappa) = (2 - \beta)(\gamma_h - 1/2) > 0$. Consequently,

$$\max\{\Pi_1^{EI}, \Pi_2^{EI}, \Pi^{NI}\} = \max\{\Pi_1^{EI}, \Pi_2^{EI}, 0\}. \tag{A.18}$$

Thus the unattained positive branch of Π^{NI} never determines the global optimum.

Regime Comparison. The preceding analysis reduces the firm's problem to a finite comparison. Using the candidate payoffs defined above, the firm's optimal payoff is

$$\Pi^* = \max\left\{\Pi_1^{EI}, \Pi_2^{EI}, \Pi_1^{DI}, \Pi_2^{DI}, \Pi_3^{DI}, \Pi_1^{SI}, \Pi_2^{SI}, \Pi_3^{SI}, \Pi_4^{SI}, \Pi^{NI}\right\},$$

where infeasible or ex-post misclassified EI, DI, and SI candidates are excluded. The term Π^{NI} already incorporates the rejection payoff through its zero branch. Moreover, by (A.18), its potentially unattained positive branch is strictly dominated by EI–2. Hence it does not affect attainment of the global optimum.

Each EI, DI, and SI candidate payoff is affine in κ, with slope equal to the negative of its expected probability of formal experimentation. The NI value is the maximum of the two affine functions $\beta(1/2-\kappa)$ and 0. Therefore, the firm's optimal payoff remains the upper envelope of finitely many affine functions of κ. □

A.2.4. Proof of Proposition 5. Fix (β, γ_h), and let $\mathcal{K}^{IS}(\alpha_1) = \{\kappa \geq 0 : \Pi^{SD}(\alpha_1, \kappa) > \Pi^{IND}(\kappa)\}$. The proof has three steps. Step 1 derives Π^{IND}, the payoff benchmark for evaluating sizing-dependent mechanisms. Step 2 compares every feasible sizing-dependent candidate with this benchmark and shows that their combined strict-gain set is either empty or a single open interval that expands with α_1. Step 3 uses this interval geometry to construct the monotone boundaries K_L, K_U and the precision threshold α^{IS}.

Step 1. The sizing-independent benchmark. By (A.8),

$$\Pi_1^{EI} = \gamma_h - \kappa, \qquad \Pi_2^{EI} = 2\gamma_h - 1 + \beta(1-\gamma_h-\kappa),$$
$$\Pi_1^{EI} - \Pi_2^{EI} = (1-\beta)(1-\gamma_h-\kappa), \qquad \kappa^E = 1-\gamma_h + \frac{2\gamma_h - 1}{\beta}.$$

Moreover, $\kappa^E - 1/2 = (2\gamma_h - 1)(2-\beta)/(2\beta) > 0$. Together with (A.18), this gives

$$\Pi^{IND}(\kappa) = \begin{cases} \Pi_1^{EI}, & 0 \leq \kappa \leq 1-\gamma_h, \\ \Pi_2^{EI}, & 1-\gamma_h \leq \kappa \leq \kappa^E, \\ 0, & \kappa \geq \kappa^E. \end{cases} \tag{A.19}$$

Step 2. Geometry of the impact-sizing region. Write

$$p_{-1}^h = \alpha_1 + \gamma_h - 2\alpha_1\gamma_h, \qquad \Delta_h = \alpha_1^2 - 2\alpha_1\gamma_h + \gamma_h,$$
$$\mathcal{N} = \alpha_1^2 + 2\alpha_1\beta\gamma_h - \alpha_1\beta - 2\alpha_1\gamma_h - \beta\gamma_h + \gamma_h, \qquad \Theta = \alpha_1 - 2\alpha_1\gamma_h(1-\gamma_h) - \gamma_h^2.$$

The lower crossings are

$$L_1 = \frac{2\alpha_1(1-\alpha_1)\gamma_h(1-\gamma_h)}{(p_{-1}^h)^2}, \qquad L_2 = \frac{\alpha_1(1-\alpha_1)(1-\gamma_h)(2\gamma_h-1)}{(2\alpha_1-1)\Theta},$$
$$L_3 = 2\alpha_1(1-\alpha_1) + (2\gamma_h - 1)\left(\frac{2}{\beta} - 1\right), \qquad U_3 = 1 - 2\alpha_1(1-\alpha_1).$$

Here and below, L_2 is used only when $\alpha_1 > \gamma_h$ and $\Theta > 0$. For the remaining upper crossings, define

$$D_1 = \alpha_1^2(\beta-2) + (1-2\alpha_1)^2(\beta-1)\gamma_h^2 - 2(2\alpha_1-1)\alpha_1(\beta-1)\gamma_h + \alpha_1,$$
$$N_2 = \gamma_h^2\{2\alpha_1[(\alpha_1-2)\beta+2] + \beta - 2\} - \gamma_h\{\alpha_1[2\alpha_1(\beta+1) - 5\beta + 2] + \beta\} + \alpha_1(\alpha_1 - \beta) + \gamma_h,$$
$$D_2 = \beta\{\alpha_1(2\gamma_h - 1) - \gamma_h\}\{\alpha_1(2\gamma_h-1) - \gamma_h + 1\},$$
$$U_1 = \frac{\alpha_1(1-\alpha_1)\gamma_h\{2\beta(1-\gamma_h) + 2\gamma_h - 1\}}{D_1},$$
$$U_2 = \frac{N_2}{D_2}.$$

Let $L_E = \max\{L_1, L_2\}$, and set

$$U_E = \begin{cases} U_1, & \mathcal{N} \le 0, \\ U_2, & \mathcal{N} \ge 0. \end{cases} \tag{A.20}$$

Because $\mathcal{N} = \Delta_h - \beta p^h_{-1}$, substitution of $\beta = \Delta_h / p^h_{-1}$ gives $U_1 = U_2$. At the same switch, DI–1, DI–2, and DI–3 all reduce to $(a_1, e_1, a_{-1}, e_{-1}) = (0, 1, \Gamma^h_{-1}, 0)$.

Define

$$\mathcal{J}_E(\alpha_1) = \begin{cases} (L_E, U_E), & \alpha_1 > \gamma_h,\ \Theta > 0,\ L_E < U_E, \\ \varnothing, & \text{otherwise}, \end{cases} \tag{A.21}$$

$$\mathcal{J}_N(\alpha_1) = \begin{cases} (L_3, U_3), & \alpha_1 > \gamma_h,\ L_3 < U_3, \\ \varnothing, & \text{otherwise}. \end{cases} \tag{A.22}$$

For $j \in \{2, 3, 4\}$, let S_{j1}, S_{j2}, S_{j0} be the crossings of SI–j with EI–1, EI–2, and zero. The required formulas are

$$S_{21} = \frac{\Delta_h(2\gamma_h - \beta)}{\Delta_h + \mathcal{N}}, \qquad S_{22} = \frac{(2-\beta)(2\gamma_h - 1)\Delta_h}{(2\alpha_1 - 1)\beta(\alpha_1 - \gamma_h)},$$
$$S_{20} = \frac{\Delta_h}{p^h_{-1}}, \qquad S_{31} = \frac{\alpha_1(1-\alpha_1)(2\gamma_h - \beta)}{(1-\beta)p^h_{-1}},$$
$$S_{32} = \frac{\alpha_1(1-\alpha_1)(2-\beta)(2\gamma_h - 1)}{(2\alpha_1 - 1)(1-\beta)(\alpha_1 - \gamma_h)}, \qquad S_{30} = \frac{\alpha_1(1-\alpha_1)\beta}{\alpha_1(1-\alpha_1) - \mathcal{N}},$$
$$S_{41} = 2\alpha_1(2\gamma_h - \alpha_1) - \beta(2\alpha_1 - 1)(2\gamma_h - 1), \qquad S_{40} = 2\Delta_h + \beta(2\alpha_1 - 1)(2\gamma_h - 1),$$
$$S_{42} = -\frac{2 + 2\alpha_1^2 - \beta - 2\alpha_1\beta - 2\gamma_h - 4\alpha_1\gamma_h + 4\alpha_1\beta\gamma_h}{2\beta - 1}.$$

The last crossing is used only when $\beta \ne 1/2$. Define the missing SI–4 attachment by

$$\mathcal{I}_4(\alpha_1) = \begin{cases} [1-\gamma_h, \min\{\kappa^E, L_3\}] \cap [S_{41}, \infty) \cap (S_{42}, \infty), & \alpha_1 > \gamma_h,\ \mathcal{N} < 0, \\ \varnothing, & \text{otherwise}. \end{cases} \tag{A.23}$$

LEMMA A.2 **(Endpoint certificate)**. *Suppose $\alpha_1 > \gamma_h$. If $\Theta \le 0$, then*

$$\begin{aligned} \mathcal{N} \ge 0 &\Longrightarrow S_{21} \ge 1 - \gamma_h, \quad S_{22} \ge \kappa^E, \quad S_{20} \le \kappa^E, \\ \mathcal{N} \le 0 &\Longrightarrow S_{31} \ge 1 - \gamma_h, \quad S_{32} \ge \kappa^E, \quad S_{30} \le \kappa^E. \end{aligned} \tag{A.24}$$

If $\Theta > 0$, then

$$\begin{aligned} &\mathcal{N} \ge 0: \quad S_{21} < 1 - \gamma_h \Rightarrow S_{21} \ge L_1, S_{22} < \kappa^E \Rightarrow \max\{1-\gamma_h, S_{22}\} > L_2, \kappa^E < S_{20} \Rightarrow S_{20} \le U_2, \\ &\mathcal{N} \le 0: \quad S_{31} > L_1, S_{32} < \kappa^E \Rightarrow \max\{1-\gamma_h, S_{32}\} > L_2, \kappa^E < S_{30} \Rightarrow S_{30} \le U_1. \end{aligned} \tag{A.25}$$

In addition, $S_{41} \ge L_1$.

Proof. Set $x = 2\alpha_1 - 1, y = 2\gamma_h - 1, b = \beta, r = 1 - xy, A = 1 + x^2 - 2xy, t = x - 2y + xy^2$. Then $0 < y < x < 1$, $r > 0$, $A > 0$, and $p^h_{-1} = \frac{r}{2}, \Delta_h = \frac{A}{4}, c := \frac{\Delta_h}{p^h_{-1}} = \frac{A}{2r} \in \left(\frac{1}{2}, 1\right), \mathcal{N} = p^h_{-1}(c - b), 4\Theta = t$. Thus $\mathcal{N} \ge 0$ is equivalent to $b \le c$, while $\mathcal{N} \le 0$ is equivalent to $b \ge c$. Let $h = (1-y)/2 = 1 - \gamma_h$. Direct substitution gives

$$\kappa^E = h + \frac{y}{b}, \qquad L_1 = \frac{(1-x^2)(1-y^2)}{2r^2}, \qquad L_2 = \frac{y(1-x^2)(1-y)}{2xt}, \tag{A.26}$$
$$S_{21} = \frac{A(1+y-b)}{2(A-br)}, \qquad S_{22} = \frac{(2-b)yA}{2xb(x-y)}, \qquad S_{20} = c,$$
$$S_{31} = \frac{(1-x^2)(1+y-b)}{2(1-b)r}, \qquad S_{32} = \frac{(1-x^2)(2-b)y}{2x(1-b)(x-y)}, \qquad S_{30} = \frac{b(1-x^2)}{2\{br - x(x-y)\}}.$$

All denominators used below have the displayed signs. In particular, $A-br \geq A/2>0$ when $b \leq c$, and $br-x(x-y) \geq (1-x^2)/2>0$ when $b \geq c$.

We first record one elementary positivity fact used on the branch $t \leq 0$. Define

$$q(x,y)=h+\frac{y}{c}-c.$$

Writing $x_0=2y/(1+y^2)$, the condition $t \leq 0$ is equivalent to $x \leq x_0$. Differentiation gives

$$\frac{\partial q}{\partial x}=\frac{\{y(1+x^2)-2x\}H(x,y)}{2A^2r^2}, \tag{A.27}$$

where $H(x,y)=1+2x^2+x^4+4y-4xy-4x^3y-8xy^2+4x^2y^2+4x^2y^3$. The first factor in the numerator of (A.27) is negative. To see that $H>0$, put $u=(x-y)/(1-y) \in (0,1)$. Then

$$\frac{H}{(1-y)^2}=(1+y)^2(1+4y)-8uy^2(1+y)+2u^2(1-y^2+2y^3)+u^4(1-y)^2.$$

The quadratic formed by the first three terms has positive leading coefficient and discriminant $-8(1+y)^2(1+4y-y^2-2y^3)<0$. Here $1+4y-y^2-2y^3>1+y>0$, because $y^2+2y^3<3y$. It is therefore positive, and so is H. Hence q decreases in x. At the right endpoint,

$$q(x_0,y)=\frac{y(1-y)(3+y+5y^2-y^3)}{2(1+y^2)(1+3y^2)}>0.$$

Consequently,

$$t \leq 0 \quad \Longrightarrow \quad \kappa^E(c)-c=q(x,y)>0. \tag{A.28}$$

Suppose first that $t \leq 0$ and $b \leq c$. From (A.26),

$$\frac{\partial S_{21}}{\partial b}=-\frac{A\{x^2-y-xy+xy^2\}}{2(A-br)^2}, \qquad x^2-y-xy+xy^2=xt-y(1-x)r<0.$$

Thus S_{21} increases in b, and $S_{21}(b)-h \geq S_{21}(0)-h=y>0$. Moreover,

$$S_{22}-\kappa^E=\frac{r}{x(x-y)}(\kappa^E-c). \tag{A.29}$$

Because κ^E decreases in b, (A.28) and $b \leq c$ imply $\kappa^E>c=S_{20}$ and $S_{22}>\kappa^E$. This proves the first line of (A.24).

Now suppose that $t \leq 0$ and $b \geq c$. We have

$$\frac{\partial S_{31}}{\partial b}=\frac{y(1-x^2)}{2(1-b)^2r}>0, \qquad S_{31}(c)=S_{21}(c)>h.$$

For the other two comparisons, define

$$P(b)=2b\{br-x(x-y)\}(\kappa^E-S_{30}).$$

Direct differentiation gives $P''(b)=2\{x^2-y-xy+xy^2\}<0$. Thus P is concave on $[c,1]$. At the endpoints, $P(c)=2c\{cr-x(x-y)\}\{\kappa^E(c)-c\}>0$, and $P(1)=y(1-x^2)>0$. Hence $P>0$, so $S_{30}<\kappa^E$. Finally,

$$S_{32}-\kappa^E=\frac{x(x-y)-br}{(b-1)x(x-y)}(\kappa^E-S_{30}). \tag{A.30}$$

Both factors in the ratio have the same negative sign. Therefore $S_{32}>\kappa^E$, proving the second line of (A.24).

It remains to prove the assertions for $t > 0$. First consider $b \le c$. Writing

$$S_{21} - L_1 = \frac{F(b)}{2r^2(A - br)}, \quad F(b) = Ar^2(1 + y - b) - (1 - x^2)(1 - y^2)(A - br),$$

we obtain

$$F'(b) = -(x - y)(xy - 1)\{-2x + y + x^2 y\} < 0.$$

Thus $F(b) \ge F(c) = yAH_1(x, y)/2$, where

$$H_1(x, y) = 2 - x + x^3 + y - 4xy - x^2 y + 2x^2 y^2.$$

This polynomial is positive. Indeed, it is convex in y. If $4x + x^2 \le 1$, it is increasing for $y \ge 0$ and $H_1(x, y) \ge 2 - x + x^3 > 0$. If $4x + x^2 > 1$, its unrestricted minimum is

$$\frac{(1 - x^2)^2(8x - 1)}{8x^2} > 0,$$

because $4x + x^2 > 1$ implies $x > \sqrt{5} - 2 > 1/8$. Therefore

$$b \le c \quad \Longrightarrow \quad S_{21} > L_1, \tag{A.31}$$

which is stronger than the first implication for $\mathcal{N} \ge 0$.

For the EI–2 endpoint, define

$$\begin{aligned} B &= 2Ar\{\kappa^E(c) - c\} = (1 - y)Ar + 4yr^2 - A^2, \\ C &= \frac{2x(x - y)t}{y}\{S_{22}(c) - L_2\} = (4r - A)t - (x - y)(1 - x^2)(1 - y). \end{aligned}$$

A direct expansion gives $2C + B = Q(x, y)$, where

$$Q = 4x - x^2 - x^4 - 7y + 3xy - 3x^2 y + x^3 y - 2y^2 + 9xy^2 - x^3 y^2 - 2x^2 y^3.$$

For fixed y, Q is concave in x, since

$$Q_{xx} = -2\{1 + 6x^2 + 3y - 3xy + 3xy^2 + 2y^3\} < 0.$$

On the interval $[x_0, 1]$,

$$\begin{aligned} Q(x_0, y) &= \frac{(1 - y)^3 y(1 + y)^2(1 + y + 5y^2 - y^3 + 2y^4)}{(1 + y^2)^4} > 0, \\ Q(1, y) &= 2(1 - y)^3 > 0. \end{aligned}$$

Hence $Q > 0$ whenever $t > 0$. If $S_{22} < \kappa^E$, (A.29) implies $\kappa^E < c$. Since $b \le c$ and κ^E decreases in b, this yields $B < 0$. Therefore $C = (Q - B)/2 > 0$, so $S_{22}(c) > L_2$. Because S_{22} decreases in b, we have $S_{22}(b) \ge S_{22}(c) > L_2$, proving the second implication for $\mathcal{N} \ge 0$.

For the zero endpoint, direct simplification gives

$$U_2 - c = \frac{yE(b)}{2b(xy - 1)(1 + xy)}, \quad E(b) = -2 - 2x^2 + 4xy + b(1 + x)(2 - x + x^2 - y - xy). \tag{A.32}$$

The coefficient of b is positive, and

$$E(c) = -\frac{(1 - x)A(2 + x + x^2 + y - xy)}{2r} < 0.$$

Indeed, $2 - x + x^2 - y - xy > 2 - 2x > 0$, because $y < x$. Thus $E(b) \le E(c) < 0$ for $b \le c$. The denominator in (A.32) is negative, so $U_2 > c = S_{20}$. This proves the third implication for $\mathcal{N} \ge 0$.

Next consider $b \ge c$. Direct simplification gives

$$S_{31} - L_1 = \frac{(1-x^2)y\{(1-x)(1+y) + b(x-y)\}}{2(1-b)r^2} > 0. \tag{A.33}$$

To handle the SI–3 EI–2 endpoint, use the polynomial P defined above. Its derivative is affine in b, with $P'(c) = H_2(x,y)/r$, and $P'(1) = x(x-y)(1+y)$, where $H_2 = x^4 + y - 2x^3y - 2xy^2 + x^2y^2 + x^2y^3$. The positivity of H_2 follows by again setting $u = (x-y)/(1-y)$:

$$\frac{H_2}{(1-y)^2} = y(1+y)\{1 + y(1-u)^2\} + 2u^3y(1-y) + u^4(1-y)^2 > 0.$$

Thus $P'(b) > 0$ on $[c,1]$. If $\kappa^E < S_{30}$, then $P(b) < 0$, and hence $P(c) < 0$. Since $S_{30}(c) = c$, this means $B < 0$. The preceding argument gives $S_{22}(c) > L_2$. Finally, S_{32} increases in b and $S_{32}(c) = S_{22}(c)$, so $S_{32} > L_2$. This proves the second implication for $\mathcal{N} \le 0$.

For the remaining zero endpoint, let $W(b) = br^2 - x^2(1+y^2) + 2xy$. Then $W(c) = \frac{(1-x^2)(1+xy)}{2} > 0$, and W increases in b. A direct common-denominator calculation gives

$$U_1 - S_{30} = \frac{(x^2-1)yG(b)}{2\{br - x(x-y)\}W(b)}, \tag{A.34}$$

where $G(b) = x(x-y)(1+y) + b\{-1 + x - y + xy - 2x^2y + 2xy^2\} + b^2(x-y)(xy-1)$. Here

$$G(c) = \frac{(1-x)^2(1+x)(2+x+x^2+y-xy)}{4(xy-1)} < 0,$$

$$G'(c) = -(1+x)(1-x+x^2-xy) < 0, \qquad G''(b) = 2(x-y)(xy-1) < 0.$$

Thus G decreases on $[c,1]$ and remains negative. Every other factor in (A.34) has the displayed sign, so $U_1 > S_{30}$. This proves the last implication for $\mathcal{N} \le 0$.

Finally, $S_{41} = \frac{(1+x)(1+2y-x)}{2} - bxy$ decreases in b. At $b = 1$, $S_{41}(1) - L_1 = \frac{yH_3(x,y)}{2r^2}$, and $H_3 = 2 - 2x + 2x^3 + y - 4xy - x^4y + 2x^2y^2$. The polynomial H_3 is convex in y. If $4x + x^4 \le 1$, it is increasing for $y \ge 0$ and $H_3(x,y) \ge 2(1-x+x^3) > 0$. If $4x + x^4 > 1$, its unrestricted minimum is $-\frac{(1-x^2)^2(1-8x+2x^2+x^4)}{8x^2} > 0$, where the final inequality follows from $1 - 8x + 2x^2 + x^4 < 2x(x + x^3 - 2) < 0$. Hence $S_{41} > L_1$, completing the endpoint certificate. $\square$

LEMMA A.3 **(Boundary monotonicity certificate)**. *On $\alpha_1 > \gamma_h$, and on each endpoint's active domain, the lower endpoints satisfy $L_1', L_2', L_3' < 0$, the upper endpoints satisfy $U_1', U_2', U_3' > 0$, and $\Theta' > 0$. Moreover, $U_1 = U_2$ when $\mathcal{N} = 0$.*

Proof. Retain the variables x, y, b, r, t from the preceding proof. Since $dx/d\alpha_1 = 2$, it is enough to differentiate with respect to x. Let $W = b - x^2 + 2xy - 2bxy - x^2y^2 + bx^2y^2$. Factoring the derivatives gives

$$\frac{\partial L_1}{\partial x} = -\frac{(x-y)(1-y^2)}{r^3}, \qquad \frac{\partial L_2}{\partial x} = -\frac{y(1-y)(x-y)r}{x^2t^2}, \qquad \frac{\partial L_3}{\partial x} = -x,$$

$$\frac{\partial U_1}{\partial x} = \frac{(1-b)(1+y)(x-y)(b+y-by)r}{W^2}, \qquad \frac{\partial U_3}{\partial x} = x, \qquad \frac{\partial \Theta}{\partial x} = \frac{1+y^2}{4}.$$

All displayed signs are immediate. For U_2,

$$\frac{\partial U_2}{\partial x} = \frac{R(b)}{br^2(1+xy)^2}, \tag{A.35}$$

where R is affine in b and its endpoint values are $R(0) = 2y\{(x-y) + xy^2(1-xy)\} > 0$, and $R(1) = (1+y)\{x - y + xy(1-y+y^2) - x^2y^3\} > 0$. For the second inequality, use $x^2y^3 < xy^3$ to bound the expression in braces by $x - y + xy(1-y) > 0$. Hence $R(b) > 0$ for every $b \in (0,1)$, and (A.35) proves that $U_2' > 0$.

Finally, $\mathcal{N} = 0$ is equivalent to $b = \Delta_h / p^h_{-1}$. Substitution into the two upper endpoints yields the common value

$$U_1 = U_2 = \frac{\gamma_h\{\alpha_1 - 2\alpha_1^2 - \gamma_h + 2\alpha_1^2\gamma_h\}}{\{-\alpha_1 - \gamma_h + 2\alpha_1\gamma_h\}\{1 - \alpha_1 - \gamma_h + 2\alpha_1\gamma_h\}}.$$

This proves the switch identity and the lemma. □

LEMMA A.4 **(Corrected candidate-envelope geometry)**. *For every $\alpha_1 \in (1/2, 1)$,*

$$\mathcal{K}^{IS}(\alpha_1) = \mathcal{J}_E(\alpha_1) \cup \mathcal{J}_N(\alpha_1) \cup \mathcal{I}_4(\alpha_1). \tag{A.36}$$

The union is empty or one open interval, is empty when $\alpha_1 \le \gamma_h$, and is nested in α_1.

Proof. Propositions A.1 and A.3 reduce the comparison to DI–1–DI–3 and SI–1–SI–4. Closed relaxations may be used because a candidate reclassified by Assumption 1 is replaced by a weakly better truthful direct mechanism. By Lemma 2, such a replacement remains sizing-dependent whenever its payoff exceeds Π^{IND}.

DI candidates. DI–1 is feasible for $\mathcal{N} \le 0$, while DI–2 and DI–3 are feasible for $\mathcal{N} \ge 0$. Their relevant comparisons are

$$\begin{aligned}
\Pi_1^{DI} - \Pi_1^{EI} &= \frac{(1-\beta)(p^h_{-1})^2}{\alpha_1(1-\alpha_1)}(\kappa - L_1), &\qquad \Pi_1^{DI} - \Pi_2^{EI} &= \frac{(2\alpha_1 - 1)(1-\beta)\Theta}{\alpha_1(1-\alpha_1)}(\kappa - L_2),\\
\Pi_2^{DI} - \Pi_1^{EI} &= p^h_{-1}(\kappa - L_1), &\qquad \Pi_3^{DI} - \Pi_2^{EI} &= \frac{(2\alpha_1 - 1)\beta\Theta}{\Delta_h}(\kappa - L_2),\\
\Pi_1^{DI} > 0 &\Longleftrightarrow \kappa < U_1, &\qquad \Pi_3^{DI} > 0 &\Longleftrightarrow \kappa < U_2.
\end{aligned} \tag{A.37}$$

When $\Theta \le 0$, the two EI–2 differences equal

$$\begin{gathered}
\frac{(2\alpha_1 - 1)(1-\beta)\Theta}{\alpha_1(1-\alpha_1)}\kappa - (1-\beta)(1-\gamma_h)(2\gamma_h - 1),\\
\frac{(2\alpha_1 - 1)\beta\Theta}{\Delta_h}\kappa - \frac{\beta\alpha_1(1-\alpha_1)(1-\gamma_h)(2\gamma_h - 1)}{\Delta_h},
\end{gathered}$$

and hence are negative. Agreement of EI–1 and EI–2 at $\kappa = 1 - \gamma_h$, together with the candidates' negative slopes, then excludes the other two benchmark branches. Thus no DI candidate gains when $\Theta \le 0$.

Suppose $\Theta > 0$. DI–1 yields (L_E, U_1) when $\mathcal{N} \le 0$. When $\mathcal{N} \ge 0$, DI–2 controls EI–1 and DI–3 controls EI–2 and zero. At $\kappa = 1 - \gamma_h$, their gains are $(1-\gamma_h)R_D / p^h_{-1}$ and $\beta(1-\gamma_h)R_D/\Delta_h$, where $R_D = \alpha_1^2 - 2\alpha_1^2\gamma_h + \gamma_h^2 - 4\alpha_1\gamma_h^2 + 4\alpha_1^2\gamma_h^2$. They have the same sign, so the two segments meet. The DI gain set is therefore exactly $\mathcal{J}_E$.

SI–1. The two necessary crossings are

$$\Pi_1^{SI} - \Pi_2^{EI} = \frac{\beta}{2}(\kappa - L_3), \tag{A.38}$$

$$\Pi_1^{SI} = \frac{\beta}{2}(U_3 - \kappa). \tag{A.39}$$

Its classification threshold lies below L_3 by $2(2\gamma_h-1)(1-\beta)/\beta>0$. To handle EI–1, let $C_S=(2\gamma_h-\beta U_3)/(2-\beta)$. Direct simplification gives

$$C_S-L_3=\frac{(1-\beta)B_S}{\beta(2-\beta)},\qquad 1-\gamma_h-C_S=\frac{B_S}{2(2-\beta)},$$
$$B_S=\beta\{(2\alpha_1-1)^2+2\gamma_h-1\}-4(2\gamma_h-1),\qquad L_1<2\alpha_1(1-\alpha_1)+2\gamma_h-1<L_3.$$

Thus $C_S>L_3$ implies $C_S<1-\gamma_h$, and a feasible DI branch covers $(L_3,C_S]$. SI–1 covers the rest of $\mathcal{J}_N$.

SI–2 and SI–3. Their payoff differences factor as

$$\begin{aligned}2\Delta_h(\Pi_2^{SI}-\Pi_1^{EI})&=(\Delta_h+\mathcal{N})(\kappa-S_{21}),\\ 2\Delta_h(\Pi_2^{SI}-\Pi_2^{EI})&=\beta(2\alpha_1-1)(\alpha_1-\gamma_h)(\kappa-S_{22}),\\ 2\Delta_h\Pi_2^{SI}&=\beta p_{-1}^h(S_{20}-\kappa),\\ 2\alpha_1(1-\alpha_1)(\Pi_3^{SI}-\Pi_1^{EI})&=(1-\beta)p_{-1}^h(\kappa-S_{31}),\\ 2\alpha_1(1-\alpha_1)(\Pi_3^{SI}-\Pi_2^{EI})&=(2\alpha_1-1)(1-\beta)(\alpha_1-\gamma_h)(\kappa-S_{32}),\\ 2\alpha_1(1-\alpha_1)\Pi_3^{SI}&=\{\alpha_1(1-\alpha_1)-\mathcal{N}\}(S_{30}-\kappa).\end{aligned}\tag{A.40}$$

If $\Theta\le 0$, the first part of Lemma A.2 makes all three branch intersections empty. Suppose $\Theta>0$. For SI–2, the three rows of the certificate show, respectively, that DI–2 beats EI–1, DI–3 beats EI–2, and DI–3 is positive whenever the corresponding SI–2 branch is profitable. For SI–3, the remaining three rows show that DI–1 beats the active benchmark branch whenever SI–3 does. Hence SI–2 and SI–3 add no points outside $\mathcal{J}_E$.

SI–4. Set $x=2\alpha_1-1$, $y=2\gamma_h-1$, and $a_4=2\gamma_h-\alpha_1-\beta(2\gamma_h-1)$. Then

$$\begin{aligned}&a_4-(1-\alpha_1)=y(1-\beta)>0, &&\Gamma_{-1}^h-a_4=-\frac{2y\mathcal{N}}{1-xy},\\ &\frac{\Delta_h}{p_{-1}^h}-\frac{1}{2}=\frac{x(x-y)}{2(1-xy)}>0, &&2(\Pi_4^{SI}-\Pi_1^{EI})=\kappa-S_{41},\\ &2(\Pi_4^{SI}-\Pi_2^{EI})=(2\beta-1)(\kappa-S_{42}), &&2\Pi_4^{SI}=S_{40}-\kappa.\end{aligned}\tag{A.41}$$

Thus SI–4 is feasible exactly when $\mathcal{N}\le 0$. Strict feasibility implies $\beta>\Delta_h/p_{-1}^h>1/2$, while at $\mathcal{N}=0$ SI–4 coincides with SI–3. Its classification condition is $\kappa\ge S_{41}$.

On the EI–1 branch, profitability gives $\kappa>S_{41}\ge L_1$. This branch is empty when $\Theta\le 0$ and is covered by DI–1 when $\Theta>0$. The retained profitable EI–2 points with $\kappa\le L_3$ are precisely $\mathcal{I}_4$. No zero-branch point can be profitable weakly below L_3, because $S_{40}-\kappa^E=-(L_3-\kappa^E)-xy(1-\beta)$. Finally, a profitable point with $\kappa>L_3$ satisfies $\kappa<S_{40}<U_3$, where $U_3-S_{40}=xy(1-\beta)>0$, and therefore lies in $\mathcal{J}_N$. This proves (A.36) for $\alpha_1>\gamma_h$.

If $\alpha_1\le\gamma_h$, DI and SI–2–SI–4 are infeasible in their sizing-dependent regimes. Any SI–1 gain would require $L_3<\kappa<U_3$, which is impossible by the low-precision calculation below. Thus (A.36) holds throughout the parameter domain.

Overlap, attachment, and openness. Both nonempty core intervals contain κ^E. First, $\kappa^E>\gamma_h>L_1$, where the second inequality follows, with $z=(\alpha_1-\gamma_h)/(1-\gamma_h)$, from

$$\gamma_h-L_1=\frac{\gamma_h(1-\gamma_h)^2}{(p_{-1}^h)^2}\left[\{4\gamma_h^2-6\gamma_h+3\}z^2-2(2\gamma_h-1)^2z+2\gamma_h(2\gamma_h-1)\right]>0.$$

The bracket has positive leading coefficient and discriminant $-4(2\gamma_h - 1) < 0$. On the two feasibility branches,

$$\frac{(2\alpha_1-1)(1-\beta)\Theta}{\alpha_1(1-\alpha_1)}(\kappa^E - L_2) = \left(-\frac{\partial \Pi_1^{DI}}{\partial \kappa}\right)(U_1 - \kappa^E),$$
$$\frac{(2\alpha_1-1)\beta\Theta}{\Delta_h}(\kappa^E - L_2) = \left(-\frac{\partial \Pi_3^{DI}}{\partial \kappa}\right)(U_2 - \kappa^E).$$

If $\mathcal{J}_E$ is nonempty, positive coefficients and $L_2 < U_E$ make both endpoint distances positive. Moreover, $\kappa^E - L_3 = U_3 - \kappa^E = (U_3 - L_3)/2$, so $\kappa^E \in \mathcal{J}_N$ whenever that interval is nonempty.

If $\mathcal{I}_4 \neq \varnothing$, then $S_{42} < L_3$, and $(2\beta-1)(L_3 - S_{42}) = \frac{1-\beta}{\beta}\{\beta x^2 - 2y + \beta y - \beta xy\}$, and $\kappa^E - L_3 = \frac{\beta x^2 - 2y + \beta y}{2\beta}$. The bracketed expression in the first equality is positive, and the numerator in the second equality exceeds it by $\beta xy > 0$. Therefore, $\kappa^E > L_3$, so $\mathcal{J}_N \neq \varnothing$ and the upper endpoint of $\mathcal{I}_4$ is L_3. Hence $\mathcal{I}_4$ attaches to $\mathcal{J}_N$, and the union in (A.36) is connected.

Because Π^{SD} and Π^{IND} are finite upper envelopes of candidate payoff functions that are continuous in κ, both envelopes are continuous in κ. Their strict-gain set $\mathcal{K}^{IS}(\alpha_1)$ is therefore open. Since it is also connected, it is either empty or a single open interval.

Low precision and nestedness. If $\alpha_1 \leq \gamma_h$, then $4\Theta = (2\alpha_1 - 1) - 2(2\gamma_h - 1) + (2\alpha_1 - 1)(2\gamma_h - 1)^2 < 0$, and $U_3 - L_3 = (2\alpha_1 - 1)^2 - (2\gamma_h - 1)\left(\frac{2}{\beta} - 1\right) < 0$. Thus all three sets in (A.36) are empty.

By Lemma A.3, L_1, L_2, L_3 decrease in α_1, whereas U_1, U_2, U_3 and Θ increase. The same lemma shows that $U_1 = U_2$ at the switch $\mathcal{N} = 0$. Therefore both $\mathcal{J}_E$ and $\mathcal{J}_N$ expand with precision.

For the remaining attachment, put $D = x - (1-\beta)y > 0$. Then $\mathcal{N}' = D > 0$, $S_{41}' = -2D < 0$, and $S_{42}' = -2D/(2\beta - 1) < 0$. Take $\alpha_1' < \alpha_1''$ and $\kappa \in \mathcal{I}_4(\alpha_1')$. If $\kappa > L_3(\alpha_1'')$, the increasing U_3 places it in $\mathcal{J}_N(\alpha_1'')$. Otherwise, the two decreasing SI–4 thresholds keep it in $\mathcal{I}_4(\alpha_1'')$ as long as $\mathcal{N}(\alpha_1'') < 0$. If $\mathcal{N}$ crosses zero, let $\alpha_0 \in (\alpha_1', \alpha_1'']$ be the unique crossing. At α_0, SI–4 equals SI–3 and still strictly beats the active EI–2 branch. SI–3 containment places κ in $\mathcal{J}_E(\alpha_0)$, hence in the nested $\mathcal{J}_E(\alpha_1'')$. This proves nestedness. $\square$

Step 3. Boundary functions and the precision threshold. When $\mathcal{K}^{IS}(\alpha_1) \neq \varnothing$, define $K_L(\alpha_1) = \inf \mathcal{K}^{IS}(\alpha_1)$ and $K_U(\alpha_1) = \sup \mathcal{K}^{IS}(\alpha_1)$. Nestedness makes K_L weakly decreasing and K_U weakly increasing. If the region is always empty, set $K_L = K_U = 0$. Otherwise, on the lower empty-precision interval, set both boundaries equal to any value between their one-sided limiting endpoints. Then $\mathcal{K}^{IS}(\alpha_1) = (K_L(\alpha_1), K_U(\alpha_1))$, and the boundaries coincide whenever $\alpha_1 \leq \gamma_h$. This proves Part (i).

Finally, fix $\kappa \in (K_L(1), K_U(1))$, where the boundary values at 1 are understood as left limits. Continuity of the candidate payoffs in α_1, together with nestedness, implies that

$$\left\{\alpha_1 \in (1/2, 1) : \kappa \in \mathcal{K}^{IS}(\alpha_1)\right\}$$

is a nonempty open upper interval. Its lower endpoint $\alpha^{IS}(\kappa; \beta, \gamma_h)$ is at least γ_h, and $\kappa \in \mathcal{K}^{IS}(\alpha_1)$ if and only if $\alpha_1 > \alpha^{IS}(\kappa; \beta, \gamma_h)$. This proves Part (ii). $\square$

A.2.5. Proof of Corollary 3.

Positive lower bound and uniform exclusion. Fix (β, γ_h). By Part (i) of Lemma A.4, $\mathcal{K}^{IS}(\alpha_1) = \varnothing$ whenever $\alpha_1 \leq \gamma_h$. Hence every precision threshold defined in Proposition 5 satisfies $\alpha^{IS}(\kappa; \beta, \gamma_h) \geq \gamma_h$. Taking the infimum over the experimentation costs in the definition of $\underline{\alpha}(\beta, \gamma_h)$ gives $\underline{\alpha}(\beta, \gamma_h) \geq \gamma_h > \frac{1}{2}$.

Now fix $\alpha_1 < \underline{\alpha}(\beta, \gamma_h)$ and any $\kappa > 0$. If $K_L(1) < \kappa < K_U(1)$, the definition of the infimum implies $\alpha_1 < \alpha^{IS}(\kappa; \beta, \gamma_h)$, so Proposition 5 implies that impact sizing is not strictly optimal. If $\kappa \notin (K_L(1), K_U(1))$, impact sizing is not strictly optimal even at precision one. Part (ii) of Lemma A.4 then rules it out at every lower precision. Thus impact sizing yields no strict payoff improvement for any $\kappa > 0$.

Limit as $\gamma_h \downarrow 1/2$. The lower bound just proved implies $\liminf_{\gamma_h \downarrow 1/2} \underline{\alpha}(\beta, \gamma_h) \geq \frac{1}{2}$. For the reverse inequality, fix $\varepsilon \in (0, 1/2)$ and choose $\alpha^\varepsilon \in (1/2, 1/2 + \varepsilon)$. Because $\alpha^\varepsilon > 1/2$, the interval $(2\alpha^\varepsilon(1-\alpha^\varepsilon),\, 1 - 2\alpha^\varepsilon(1-\alpha^\varepsilon))$ is nonempty. Choose κ^ε in this interval.

By (A.38)–(A.39), the SI–1 core interval at $(\alpha^\varepsilon, \gamma_h)$ is

$$\left(2\alpha^\varepsilon(1-\alpha^\varepsilon) + (2\gamma_h - 1)\left(\frac{2}{\beta} - 1\right),\, 1 - 2\alpha^\varepsilon(1-\alpha^\varepsilon)\right).$$

As $\gamma_h \downarrow 1/2$, the lower endpoint converges to $2\alpha^\varepsilon(1-\alpha^\varepsilon)$, whereas the upper endpoint is unchanged. Therefore, for every $\gamma_h > 1/2$ sufficiently close to $1/2$, we have $\gamma_h < \alpha^\varepsilon$ and $\kappa^\varepsilon \in \mathcal{J}_N(\alpha^\varepsilon)$. Equation (A.36) then implies that impact sizing is strictly optimal at $(\alpha^\varepsilon, \kappa^\varepsilon)$.

By Part (ii) of Lemma A.4, impact sizing is then also strictly optimal at $\alpha_1 = 1$. Hence $\kappa^\varepsilon \in (K_L(1), K_U(1))$ and its precision threshold is well-defined. Proposition 5 gives $\underline{\alpha}(\beta, \gamma_h) \leq \alpha^{IS}(\kappa^\varepsilon; \beta, \gamma_h) < \alpha^\varepsilon < \frac{1}{2} + \varepsilon$. Because ε is arbitrary, $\limsup_{\gamma_h \downarrow 1/2} \underline{\alpha}(\beta, \gamma_h) \leq \frac{1}{2}$. Combining the liminf and limsup inequalities proves $\lim_{\gamma_h \downarrow 1/2} \underline{\alpha}(\beta, \gamma_h) = \frac{1}{2}$.

Limit as $\gamma_h \uparrow 1$. For every $\gamma_h \in (1/2, 1)$, $\gamma_h \leq \underline{\alpha}(\beta, \gamma_h) \leq 1$. Letting $\gamma_h \uparrow 1$ and applying the squeeze theorem gives $\lim_{\gamma_h \uparrow 1} \underline{\alpha}(\beta, \gamma_h) = 1$. This completes the proof. $\square$

A.2.6. Proof of Proposition 6. Fix (β, γ_h) and let

$$\kappa \in \left(\kappa^E(\beta, \gamma_h), \frac{\kappa^E(\beta, \gamma_h)}{\gamma_h}\right).$$

All equilibrium classifications below use Assumption 1. We first show that the uninformative-sizing benchmark does not induce effort. We then construct an effort-inducing, sizing-dependent policy at $\alpha_1 = 1$ and extend the comparison to sufficiently precise sizing. The effort incentive is strict in the lower-cost subregion and binds in the upper-cost subregion.

The uninformative-sizing benchmark. From (A.8), the two relevant effort-inducing payoffs are $\Pi_1^{EI} = \gamma_h - \kappa$ and $\Pi_2^{EI} = \beta\{\kappa^E(\beta, \gamma_h) - \kappa\}$. Moreover, $\kappa^E(\beta, \gamma_h) - \gamma_h = \frac{(2\gamma_h - 1)(1-\beta)}{\beta} > 0$. Because $\kappa > \kappa^E(\beta, \gamma_h)$, both EI payoffs are strictly negative. We also have $\kappa > \gamma_h > 1/2$, so $\Pi^{NI} = 0$. Therefore, rejection is optimal under the uninformative-sizing benchmark, and the benchmark does not induce effort.

The no-effort envelope at perfect precision. Now consider $\alpha_1 = 1$. At this limit, SI–3 and SI–4 are infeasible because their feasibility condition would require $(1-\beta)(1-\gamma_h) \leq 0$. SI–1 and SI–2 have the same limiting payoff, $\Pi_1^{SI}(1, \kappa) = \Pi_2^{SI}(1, \kappa) = \frac{\beta(1-\kappa)}{2}$. The remaining no-effort candidates are either infeasible or dominated, as established in Proposition A.3. Hence the no-effort envelope at $\alpha_1 = 1$ is $\max\left\{\frac{\beta(1-\kappa)}{2}, 0\right\}$.

We now divide the experimentation-cost interval into two subregions.

Lower-cost subregion. Suppose $\kappa \le 1/\gamma_h - 1$. Consider the limiting DI–2 policy $(a_1, e_1, a_{-1}, e_{-1}) = (0, 1, 0, 0)$. It experiments after a positive report and rejects after a negative report. At $\alpha_1 = 1$, a manager with a positive signal strictly prefers the positive report. A manager with a negative signal is indifferent between the two reports, but the firm strictly prefers the negative report because it avoids an unproductive experiment. Assumption 1 therefore selects truthful reporting.

The manager's payoff from effort and truthful reporting, net of effort cost, exceeds the best no-effort payoff by $\gamma_h - \beta\left(\gamma_h - \frac{1}{2}\right) - \frac{1}{2} = (1-\beta)\left(\gamma_h - \frac{1}{2}\right) > 0$. Thus effort is strictly induced. The effort outcome in this subregion therefore does not depend on the effort tie-breaking convention. Assumption 1 is used here only to resolve the negative-signal manager's reporting indifference. The firm's payoff and its advantage over the no-effort sizing-dependent envelope are

$$\Pi_2^{DI}(1,\kappa) = \gamma_h(1-\kappa) > 0,$$
$$\Pi_2^{DI}(1,\kappa) - \Pi_1^{SI}(1,\kappa) = \frac{(2\gamma_h - \beta)(1-\kappa)}{2} > 0.$$

The case condition implies $\kappa < 1$, and $2\gamma_h > \beta$. Hence DI–2 strictly dominates both the no-effort envelope and the sizing-independent envelope.

Upper-cost subregion. Suppose instead that $\kappa > 1/\gamma_h - 1$. Consider the limiting DI–3 policy $(a_1, e_1, a_{-1}, e_{-1}) = (1-\beta, \beta, 1-\beta, 0)$. At $\alpha_1 = 1$, a manager with a positive signal strictly prefers the positive report. Following a negative signal, the manager is indifferent between reports, but the firm strictly prefers the negative report because it avoids an unproductive experiment. Truthful reporting is therefore selected.

The manager obtains $1 - \beta/2$ both from effort with truthful reporting, net of effort cost, and from the best no-effort strategy. Thus the effort constraint binds. The firm's payoff under effort is $\Pi_3^{DI}(1,\kappa) = \beta\left\{\kappa^E(\beta,\gamma_h) - \gamma_h\kappa\right\} > 0$, where the inequality follows from $\kappa < \kappa^E(\beta,\gamma_h)/\gamma_h$. Its payoff advantage over the no-effort sizing-dependent envelope is

$$\Pi_3^{DI}(1,\kappa) - \Pi_1^{SI}(1,\kappa) = (2\gamma_h - 1)\left[1 - \frac{\beta(1+\kappa)}{2}\right].$$

This difference is strictly positive because $\frac{2}{\beta} - 1 - \frac{\kappa^E(\beta,\gamma_h)}{\gamma_h} = \frac{1-\beta}{\beta\gamma_h} > 0$, which implies $\kappa < \kappa^E(\beta,\gamma_h)/\gamma_h < 2/\beta - 1$. The same payoff difference is the firm's advantage from effort with truthful reporting over the manager's tied no-effort strategy. Because the manager is indifferent, the classification of this policy as effort inducing relies on Assumption 1. Under that assumption, effort and truthful reporting are selected. The resulting DI–3 payoff strictly exceeds both the no-effort and sizing-independent envelopes.

Extension to sufficiently precise sizing. In both subregions, an effort-inducing, sizing-dependent policy strictly dominates every no-effort and sizing-independent alternative at $\alpha_1 = 1$. It remains to verify that these comparisons persist below perfect precision.

The DI–2 and DI–3 feasibility threshold converges to 1 as $\alpha_1 \uparrow 1$. Since $\beta < 1$, both candidates remain feasible for all α_1 sufficiently close to 1. Moreover, $\Gamma_{-1}(\gamma_h) \to 0$, so the firm's strict preference for truthful reporting after a negative signal also persists. In the lower-cost subregion, the manager's effort incentive remains strict. In the upper-cost subregion, the effort constraint remains binding, while the firm's strict preference for effort persists. Assumption 1 therefore continues to select effort. Finally, all candidate payoffs and the competing finite envelopes are continuous in α_1.

The upper-cost effort outcome can also be approximated by policies with a strict effort incentive. Starting from DI–3, increase e_1 by a sufficiently small $\varepsilon > 0$, reduce a_1 by the same amount, and set $a_{-1} = 1 - (1 - \Gamma^h_{-1})(e_1 + \varepsilon)$, and $e_{-1} = 0$. This adjustment preserves feasibility and the negative-signal truth-telling equality. It increases the left-hand side of the effort constraint by $\left[\alpha_1 - (2\alpha_1 - 1)\Gamma^h_{-1}\right]\varepsilon > 0$, so effort becomes strictly preferable. The firm's payoff converges to the DI–3 payoff as $\varepsilon \downarrow 0$. Thus the exact DI–3 policy uses Assumption 1, but its effort outcome and payoff can be approached by policies with a strict effort incentive.

Therefore, there exists $\alpha^C(\kappa; \beta, \gamma_h) \in (1/2, 1)$ such that, whenever $\alpha_1 > \alpha^C(\kappa; \beta, \gamma_h)$, an effort-inducing, sizing-dependent policy strictly dominates every policy that either does not induce effort or does not use sizing. The optimal policy consequently induces both sizing and effort. Since the uninformative-sizing benchmark induces no effort, sizing complements effort. □

A.2.7. Proof of Proposition 7. The proposition is an existence result under Assumption 1. We exhibit an interior parameter vector at which the selected optimal outcome under the uninformative-sizing benchmark involves effort, whereas the selected optimal policy with informative sizing is SI. We then extend these selected outcomes to an open parameter region by continuity. Consider $(\beta, \gamma_h, \alpha_1, \kappa) = \left(\frac{9}{10}, \frac{11}{20}, \frac{3}{4}, \frac{13}{25}\right)$.

Step 1: The uninformative-sizing benchmark. When $\alpha_1 = 1/2$, the sizing signal contains no information about quality, so report-contingent policies cannot improve on the sizing-independent envelope. Substituting the displayed values into (A.8) and (A.18) gives $\Pi_1^{EI} = \frac{3}{100}$, $\Pi_2^{EI} = \frac{37}{1000}$, and $\Pi^{NI} = 0$. Therefore, EI–2 gives the firm the highest payoff under the uninformative-sizing benchmark. At EI–2, however, the manager is indifferent between effort and no effort. The firm's payoff from selecting effort exceeds its payoff from selecting no effort by $(2 - \beta)(\gamma_h - 1/2) = 11/200 > 0$. Assumption 1 therefore selects effort. Thus, the benchmark effort outcome relies on the maintained equilibrium-selection convention.

Step 2: The optimal policy with informative sizing. At $\alpha_1 = 3/4$, consider SI–4 in (A.14). Substitution gives $(a_1, e_1, a_{-1}, e_{-1}) = \left(0, 1, \frac{13}{50}, 0\right)$. This policy is feasible and sizing-dependent. Under no effort, the positive- and negative-signal truth-telling margins are 49/100 and 1/100, respectively, so truthful reporting is strict.

It remains to verify that the manager does not exert effort. Relative to truthful reporting without effort, the manager's payoff losses from effort followed by T, P, N, and S are, respectively, 7/500, 0, 29/100, and 69/250. Thus, the only binding no-effort constraint is the deviation to effort followed by pooling on the positive report. At this tie, the firm's payoff difference is $\widehat{\Pi}_l^T - \widehat{\Pi}_h^P = \frac{1}{50} > 0$. The manager is therefore indifferent between truthful reporting without effort and effort followed by pooling on the positive report. Because the firm's payoff is higher under truthful reporting without effort, Assumption 1 selects that strategy. Hence SI–4 is classified as SI under the maintained selection convention, with selected payoff $\Pi_4^{SI} = 1/20$.

Step 3: Global optimality and an open region. By Propositions A.1, A.2, and A.3, only the finite set of candidates characterized above needs to be considered. The other feasible candidates have selected payoffs

$$\left(\Pi_1^{EI}, \Pi_2^{EI}, \Pi_1^{DI}, \Pi_1^{SI}\right) = \left(\frac{3}{100}, \frac{37}{1000}, \frac{6461}{150000}, \frac{189}{4000}\right),$$

all of which are strictly below $1/20$. DI–1 and SI–3 coincide at this parameter vector. Because $\kappa > 2\Gamma^h_{-1}(1 - \Gamma^h_{-1})$, the principal-preferred rule selects truthful reporting after a negative signal. The effort-selection test in (A.16) then assigns this policy to DI. The remaining candidates cannot enter their stated regimes. DI–2 violates its effort constraint by $7/475$, DI–3 violates nonnegativity because $a_1 = -56/115$, and SI–2 violates feasibility because $e_1 = 171/115 > 1$. The NI value is zero. Therefore, SI–4 is the unique optimal policy with informative sizing.

The candidate policies, their payoffs, and all relevant incentive and selection margins are continuous in the parameters near the displayed vector. All firm-payoff comparisons and reporting margins established above are strict. Although the manager's effort constraint under EI–2 and the no-effort constraint against positive-report pooling under SI–4 bind, the firm has a strict preference between the tied strategies in each case. These principal-selection margins therefore retain their signs locally. Under Assumption 1, throughout an open neighborhood of the displayed vector, the uninformative-sizing benchmark continues to select effort under EI–2, whereas SI–4 remains uniquely optimal with informative sizing and continues to select no effort.

The two selected effort outcomes can be approached using policies with strict effort incentives. Under the uninformative-sizing benchmark, replace the EI–2 policy $(1-\beta, \beta)$ with $(1-\beta-\varepsilon, \beta+\varepsilon)$. Effort then yields the manager a strict payoff gain of $\varepsilon(\gamma_h - \gamma_l) > 0$.

Under informative sizing, replace the SI–4 policy $\left(0, 1, \frac{13}{50}, 0\right)$ with $\left(0, 1, \frac{13}{50} + \varepsilon, 0\right)$. The manager's payoff from truthful reporting without effort then exceeds the payoff from effort followed by pooling on the positive report by $\varepsilon/2$. Because all other reporting and effort inequalities have strict margins at the displayed parameter vector, they remain satisfied for sufficiently small $\varepsilon > 0$. In both cases, the firm's payoff converges to the reported value as $\varepsilon \downarrow 0$. These perturbations do not change the fact that the exact boundary policies are classified using Assumption 1.

Because the displayed vector is interior to the maintained parameter domain, this neighborhood contains an open product of intervals satisfying the bounds in the proposition. Taking the endpoints of these intervals as $\underline{\beta}_s, \bar{\beta}_s$, $\underline{\gamma}_s, \bar{\gamma}_s$, $\underline{\alpha}_s, \bar{\alpha}_s$, and $\underline{\kappa}_s, \bar{\kappa}_s$ proves that sizing substitutes for effort throughout a nonempty open region. $\square$

A.3. Normalization of the Experiment-Based Policy

We show that, within the class of post-experiment adoption rules that are monotone in the experimental outcome, dependence on the manager's report can be eliminated without changing the manager's incentives or reducing the firm's payoff.

DEFINITION A.1 (MONOTONE POST-EXPERIMENT ADOPTION RULE). A history-dependent post-experiment adoption rule is *monotone* if $\tau_2(r, 1) \geq \tau_2(r, -1)$ for every $r \in \{-1, 1\}$.

LEMMA A.5 **(Normalization of Monotone Post-Experiment Rules)**. *Suppose that s_1 and s_2 are conditionally independent given q, that $\alpha_2 \in [1/2, 1]$, and that $\kappa \geq 0$. For every feasible policy with a monotone history-dependent post-experiment adoption rule, there exists another feasible policy whose post-experiment adoption rule is report-independent and satisfies $\widetilde{\tau}_2(1) = 1$, and $\widetilde{\tau}_2(-1) = 0$. The transformed policy preserves the manager's expected implementation probability after every report and posterior belief and weakly increases the firm's payoff. Consequently, it preserves all reporting, effort, and participation constraints.*

Proof. Fix a report $r \in \{-1,1\}$. Let a_r and e_r be the probabilities of direct adoption and experimentation following report r, and write

$$d_{r,+} = \tau_2(r,1), \qquad d_{r,-} = \tau_2(r,-1).$$

Monotonicity implies $d_{r,+} \geq d_{r,-}$.

Consider any posterior probability Γ that the project is high quality upon reaching this report branch. Conditional independence implies that the probabilities of the two experimental outcomes are $p_+(\Gamma) = \alpha_2\Gamma + (1-\alpha_2)(1-\Gamma)$, and $p_-(\Gamma) = 1 - p_+(\Gamma)$. The corresponding contributions to the firm's expected implementation payoff are $w_+(\Gamma) = \alpha_2\Gamma - (1-\alpha_2)(1-\Gamma)$, and $w_-(\Gamma) = (1-\alpha_2)\Gamma - \alpha_2(1-\Gamma)$, where $w_+(\Gamma) + w_-(\Gamma) = 2\Gamma - 1$.

The transformation removes experimentation that does not affect the final implementation decision. Any adoption probability common to both experimental outcomes can be shifted to direct adoption, while any rejection probability common to both outcomes can be shifted to immediate rejection. Only the outcome-dependent component requires experimentation. The transformation therefore preserves implementation probabilities conditional on quality while weakly reducing experimentation costs.

For report r, define the transformed sizing-based probabilities by

$$\widetilde{a}_r = a_r + e_r d_{r,-}, \qquad \widetilde{e}_r = e_r(d_{r,+} - d_{r,-}),$$

and use the canonical post-experiment rule $\widetilde{\tau}_2(1) = 1$ and $\widetilde{\tau}_2(-1) = 0$.

For every posterior Γ, the manager's expected implementation probability following report r is preserved because $\widetilde{a}_r + \widetilde{e}_r p_+(\Gamma) = a_r + e_r\left[d_{r,+}p_+(\Gamma) + d_{r,-}p_-(\Gamma)\right]$. The transformation therefore preserves the manager's payoff from every report under every effort choice. Manipulation and effort costs are unchanged, so all reporting, effort, and participation constraints are preserved.

Feasibility is also preserved. In particular, $\widetilde{a}_r + \widetilde{e}_r = a_r + e_r d_{r,+} \leq a_r + e_r \leq 1$, and monotonicity ensures that $\widetilde{a}_r, \widetilde{e}_r \geq 0$.

It remains to compare the firm's payoff. Using $w_+(\Gamma) + w_-(\Gamma) = 2\Gamma - 1$, the transformed branch payoff minus the original branch payoff equals $e_r\kappa\left(1 - d_{r,+} + d_{r,-}\right) \geq 0$. The inequality follows from $d_{r,+} \leq 1$, $d_{r,-} \geq 0$, and $\kappa \geq 0$. Thus the transformation weakly increases the firm's payoff for every report branch and posterior belief. When $\kappa > 0$ and experimentation occurs, the improvement is strict unless the original post-experiment rule is already canonical.

Applying this transformation separately after each report produces a feasible policy with the same canonical post-experiment rule in both report branches. The resulting rule therefore depends only on s_2, preserves all of the manager's incentives, and weakly increases the firm's payoff. This proves the lemma. $\square$