

# Labor Cost Free-Riding in the Gig Economy

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## Abstract

We propose a theory of gig economies in which workers participate in a shared labor pool utilized by multiple firms. Firms must make a trade-off when setting pay rates. High pay rates are necessary to maintain a large worker pool and thus reduce the likelihood of lost demand, but they also lower profit margins. We show that workers will accept any job paying more than their reservation wage rate, yet larger firms pay more than smaller firms in the resulting pay equilibrium. This leads to a strong free-riding effect; specifically, firms smaller than a critical size pay the minimal rate possible (the workers' reservation wage), while all firms larger than the critical size earn the same total profit regardless of size, implying strong diseconomies of scale in labor costs. Nevertheless, we show that the formation of a gig economy requires the existence of a firm large enough to support a worker pool on its own. We then analyze the role of demand competition. Under perfect price competition, market collapse is the only equilibrium. However, when at least one firm has a sufficiently large base of loyal customers, market collapse is avoided, and free-riding occurs in the pay equilibrium. Comprehensive simulations confirm the robustness of these results to various model assumptions. Our results offer insights into critical strategic decisions for gig-economy firms, such as the rationality of entry by small firms, the advantages and defensibility of scale, and the payoff firms can expect from gaining market share.

**Key words:** Gig economy, economies of scale, wage equilibrium, free-riding, queuing

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# 1 Introduction

The gig economy has grown rapidly over the past decade. Between 2010 and 2020, the share of gig workers increased to 15% of the total US labor force (Iacurci 2020) and two-thirds of major US companies now report using freelance contract workers (Arruda 2020). Gig workers provide labor to many industries, with ride-hailing, food delivery, casual labor, and similar online platform businesses representing the fastest-growing sectors. The leading companies in this group – Uber, Lyft, DiDi, Doordash, and Airbnb – achieved lofty valuations, often exceeding the value of long-standing incumbents. In short, gig economy companies and gig work are now prominent features of the modern economy.

Yet, gig economy work is fundamentally different from traditional employment. Gig jobs do not involve long-term commitments between a firm and its workers, and the unit of contracted work is small, often a single task or job (e.g., providing a ride or making a delivery). Gig workers have the flexibility to work as much or as little as they want. They have the freedom to sign up and work for multiple firms to improve their earnings – often switching between different platforms in real-time (referred to as “multi-apping”). For example, it is common for ride-share drivers in the U.S. to accept rides from both Lyft and Uber and to switch to food delivery apps during lunch and dinner time<sup>1</sup>. As estimated by M. K. Chen, Rossi, and Chevalier 2019, this flexibility is valuable for workers, providing twice the surplus they would otherwise derive from less flexible work.

Multi-apping benefits workers in many ways. It reduces idle time, increasing utilization and hourly earnings. It also helps to diversify the risk of significant downtime due to unexpected events such as app outages or being banned from a particular platform. Indeed, the threat of losing the ability to multi-app has been cited as a key reason for the successful passage of Proposition 22 in California, a ballot measure jointly supported by Uber, Lyft, Postmates, DoorDash, and Instacart to exempt app-based gig workers from classification as employees<sup>2</sup>. Multi-apping is often encouraged by firms too, especially new entrants to a market. For example, when Juno (which at one point became the second largest ride-hailing company in New York City (NYC) by driver count) entered the NYC ride-hailing market, it paid Uber drivers \$25/week to keep the Juno app open when they drove for Uber (Solomon 2016). Although not all gig workers multi-app, the practice is increasing. Using bank data that tracked workers’ earnings up to 2018, JPMorgan Chase Institute (Farrell, Greig, and Hamoudi 2018) reports a steady increase in the percentage of gig transportation workers that are multi-apping, from approximately 2.5% in 2012 to over 20% in 2018. In short, multi-apping is an important feature of gig work and in many ways defines what it means to be a “gig worker”.

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<sup>1</sup>*Don't Put All Your Delivery Eggs in One Basket – Keep Your Earnings Options Open.* The Entre Courier. <https://entrecourier.com/2019/07/05/know-your-revenue-options/>

<sup>2</sup>See <https://yeson22.com/> for example

At a market level, when gig workers multi-app, they form a shared pool of labor that multiple firms can access. The existence of this shared labor pool raises interesting questions for both workers and firms. For one, when workers generate earnings from multiple platforms, how do they decide whether or not to work and which jobs to accept while working? In turn, knowing that workers have the flexibility to take jobs from other firms, how does a firm set its pay rate? How is the cost of maintaining the worker pool shared among firms and how is such a pool formed in the first place? Lastly, what is the impact of a shared worker pool on demand-side competition among firms? Our theory provides precise answers to these questions, yielding important consequences for firm-level strategy and insights into the formation, structure, and potential regulations of the gig economy.

## 1.1 Model overview and key results

We model a gig economy as a set of  $N$  firms that use workers from a common pool. Each firm receives stochastic arrivals of potential demand (jobs) and seeks to maximize its profit. Firm profit depends on several components. First, each job served generates revenue from the customer and incurs labor costs, which the firm pays to the worker who completes the job. The number of jobs a firm serves depends on both demand and supply: on the demand side, the arrival rate of jobs follows a demand curve and is determined jointly by the pricing decisions of all firms; on the supply side, if a job arrives but no worker is available, it is assumed to be lost, an outcome more likely when the worker pool is smaller. In addition, workers make rational decisions about which jobs to accept, requiring firms to set pay high enough to attract workers. As a result, a firm’s profit-maximization problem involves carefully balancing these competing factors. We model these decisions as a multistage game with sequential interactions between firms and workers. The sequence of events is illustrated in Fig. 1.

We solve the game using backward induction. The summary of results in this section mirrors this order. We start with the workers’ decisions, including the acceptance decision and the worker arrival rate. Next, we present the firms’ decisions, starting with the worker pay equilibrium, followed by demand realization and the pricing decision.

**Worker decisions and equilibrium pool size** In Section 2.1, we focus on the decisions of the workers in the last two stages in Fig. 1. The worker pool is modeled as a simple M/M/1 queue; workers join the pool at rate  $\lambda$  (the participation rate) and leave it when they accept a job. We first study the worker’s individual-level job acceptance policy while participating in the pool; that is, when a worker is offered a job by firm  $i$  with a corresponding pay  $p_i$ <sup>3</sup>, whether to accept or reject it. The participation rate  $\lambda$  is then determined by the equilibrium entry of workers. Workers seek to maximize their expected

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<sup>3</sup>Note that  $p_i$  represents the pay per job hour. That is, workers are compensated at the rate  $p_i$  while they complete a job. This pay does not cover their idle time while waiting for jobs.

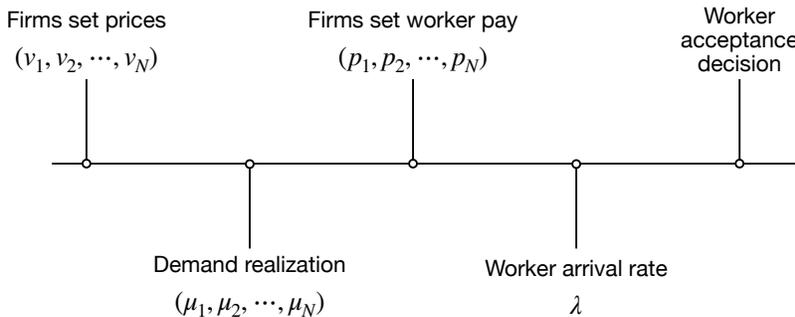


Figure 1: Sequence of events

hourly earnings subject to earning at least a reservation wage rate  $w_0$  (their outside option), which is assumed to be identical for all workers. If workers earn more than  $w_0$  from participating, more workers will participate, and  $\lambda$  increases; if they earn less than  $w_0$ , workers stop participating, and  $\lambda$  decreases. Hence, in equilibrium, workers earn just  $w_0$ . For both these stages, we assume that the number of workers is large; hence, the equilibria that result in these stages are rational expectations equilibria.

We show that workers may accept a firm’s job even if it pays less than other firms because there is an opportunity cost from rejecting a job and waiting (without pay) for a better job. In fact, under a simple and plausible model of worker behavior, we show that it is optimal for workers to accept any job paying  $w_0$  or more. This means that some firms can potentially pay workers less than other firms. However, if a firm pays less to increase its profit per job, workers earn less on average, reducing the equilibrium participation rate  $\lambda$ . A reduction in  $\lambda$ , in turn, leads to a smaller pool of available workers. This decreases the job completion rate for all firms, as firms are more likely to find the worker queue empty. Firms must therefore balance between paying less to increase margin and paying more to maintain a high job completion rate.

**Worker pay and free-riding.** Section 3.1 studies the resulting equilibrium among firms. Anticipating the worker job acceptance policy and the equilibrium arrival rate  $\lambda$ , and given the job arrival rates  $\mu_i$  and prices  $v_i$  for all firms  $i$ , each firm  $i$  chooses its worker pay  $p_i$ . We characterize the resulting multistage equilibria and show that in any equilibrium with a positive worker arrival rate (i.e., when a market can form), there are two categories of firms: “Smaller” firms <sup>4</sup> *free-ride* and offer worker pay at the minimum level  $w_0$ . The remaining “large” firms pay strictly more than the minimum  $w_0$  and hence bear the cost of maintaining the worker pool. Moreover, all large firms have a per job profit margin that is inversely proportional to their job arrival rate. This means that they all earn the same

<sup>4</sup>The size of the firm refers to the potential earnings if all their job requests are fulfilled. For example, “smaller” firms may have a lower job demand rate or lower expected revenue for completed jobs

profit regardless of size, producing a dramatic *diseconomy of scale* in worker costs.

This labor cost diseconomy of scale has important implications. For one, it contradicts the conventional wisdom about the role of scale and competitive advantage in ride-hailing. In fact, in the prospectus for its initial public offering, Uber highlighted their “massive network” and “creating a liquidity network effect” as two of its most important competitive advantages. They also stated that: “Generally, for a given geographic market, we believe that the operator with the larger network will have a higher margin than the operator with the smaller network.”<sup>5</sup> While there are well-known spatial density effects in ride-hailing systems that create benefits from increased scale (see Nikzad 2020), our results point to an off-setting – and potentially more powerful – diseconomy of scale in ride-hailing labor costs. Our finding also has implications for the defensibility of scale, showing that small firms that enter may enjoy significant labor cost advantages against large incumbents.

Our results are a form of the classical free-rider problem as described in Olson 1965, Stigler 1974, Groves and Ledyard 1977, etc., in which idle workers waiting for jobs can be viewed as a shared public resource that is nonexcludable (due to the flexible work arrangement) and partially nonrivalrous (due to the stochasticity in the job arrival), creating the possibility for small firms to access this collective resource without contributing to the aggregate compensation for workers’ idle time.

**Market formation.** Section 3.2 then studies the existence and uniqueness of the worker pay subgame, with a special focus on when an equilibrium with a positive worker arrival rate exists. Results show that the presence of a sufficiently large firm is needed to jump-start this gig economy, because small firms would not likely be able to coordinate to create a viable worker pool. However, once a large firm has created a worker pool, smaller firms can then use it at a low cost. In a stylized way, this finding is consistent with the history of the gig economy, where large firms like Uber in ride-hailing spent massively to establish initial pools of workers, opening the door for other gig businesses to flourish.

**Pricing and competition.** Section 4 then studies how the free-riding effect manifests itself under different demand models. We look at a spectrum of demand settings, including two special cases (non-competitive demand and perfect competition) and one general case (imperfect competition). We find that, while perfect competition always leads to market collapse (zero worker arrival rate), as long as one of the firms has a sufficiently large base of loyal customers, then market collapse is never an equilibrium. These findings validate the robustness of the free-riding effect under various demand conditions.

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<sup>5</sup>Form S-1, Uber Technologies, Inc., <https://www.sec.gov/Archives/edgar/data/1543151/000119312519103850/d647752ds1.htm>.

**Robustness and simulation.** We believe our results generalize beyond our model assumptions. As long as some workers are willing to multi-app (and they generally have good reasons to), these workers can be thought of as a public good. This means that small firms can decrease pay as long as the workers are still willing to work for them, since the larger firms maintain the public good. However, larger firms cannot afford to reduce pay because doing so significantly impacts the size of the worker pool and the firm’s profit. Of course, many other effects may influence worker pay in practice; we only claim that the free-riding effect we uncovered is one of them. Still, we test the robustness of our assumptions in Section 5, introducing a comprehensive numerical framework to evaluate platform equilibria. We find that smaller firms pay workers less than larger firms in all settings, including when we significantly change the order of events or add the possibility of firms having dedicated workers.

## 1.2 Related literature

Empirical evidence suggests a positive relationship between wages and firm size (see Moore 1911, Brown and Medoff 1989, Idson and Oi 1999 for examples) and multiple theories have been proposed to explain this effect. For example, Idson and Oi 1999 shows that larger firms have higher capital-labor ratios and are early adopters of new technologies; and thus have a higher performance standard for workers. As a result, to properly incentivize workers and prevent skilled workers from taking other job offers, larger firms choose to compensate them with higher wages. Other explanations include that larger firms offer inferior work conditions, have a greater ability to pay higher wages, and are less able to monitor workers (Brown and Medoff 1989). Despite similar predictions about the pay difference between large and small firms, the driving forces in these explanations are qualitatively different from ours. In this literature, the notion of a shared labor pool does not exist; workers are full-time employees who only work for one firm at a time. Firms’ wage decisions are thus only driven by their own characteristics, like hiring standards or working environment. In contrast, our setting is specific to the modern gig economy. Workers are homogeneous, paid per job, and accessible to all firms. Since workers’ participation decisions depend on their aggregate earnings across all gig-economy firms, which are generated by all firms jointly, there is an inherent externality in firms’ wage-setting decisions. These externalities in wage-setting are central to our theory but not considered by the previous literature.

Our model can be viewed as a search model since workers and jobs cannot be matched immediately and market friction (workers’ waiting time) plays an important role in both the worker strategies and firm strategies. Rogerson, Shimer, and Wright 2005 provide a comprehensive review of the search-theoretic models, which we largely base our discussions on. Our paper is particularly related to the search models of wage dispersion and the directed search models. The wage dispersion literature aims to explain why homogeneous workers may receive different pay. For example, Burdett and Mortensen 1998 show that when workers can receive and accept job offers both while being unemployed and employed,

wage dispersion arises as the unique equilibrium, even when the firms and the workers are both homogeneous. Moreover, in equilibrium, there is a positive relationship between a firm’s pay and its worker supply. The key driving force in Burdett and Mortensen 1998 is the workers’ ability to accept jobs while being employed; due to this ability, workers are motivated to quit and switch to a better-paying job whenever they receive one. Therefore, a firm faces a trade-off between the profit margin and the workers’ quitting rate. However, once this ability is removed, the wage equilibrium in Burdett and Mortensen 1998 degenerates; Workers will accept any job that pays no less than their reservation wage, and all firms pay exactly the reservation wage (an outcome known as the “Rothschild Critique” (Rothschild 1978)). The fundamental reason is that, in Burdett and Mortensen 1998, the concept of a shared worker pool is non-existent; in their model, workers do not incur the idleness of waiting for a job, but switch between unemployment and employment directly. Thus, the supply of employed workers is fixed as long as the pay is no less than the reservation wage; firms pay differently only to attract workers from their competitors, but once workers cannot search while employed, there is no incentive to pay more. Albrecht and Axell 1984 derive wage dispersion by considering workers who are heterogeneous in their value of leisure time; Van den Berg and Van Vuuren 2010 shows that when firms are heterogeneous in productivity, multiple equilibria arise. In contrast, we propose a new mechanism of wage dispersion which is driven by a shared labor pool among gig-economy firms. The fact that the worker pool is shared critically depends on the unique characteristics of the gig economy that workers are free to switch among firms on a job-by-job basis and that the size of the worker pool is positively related to the firms’ pay. Because of this shared labor pool, large firms’ pay rates have a higher return in increasing the size of the shared pool; wage dispersion arises in equilibrium as a result of this asymmetric impact that firms have on the labor pool.

Our model also has an important connection to the directed search literature, such as Moen 1997. This literature assumes that firms post wages *ex ante* and workers try to apply to firms with the most attractive terms. A key difference between the directed search models and our model is the shared labor pool. In the directed search literature, each firm typically has its own queue (worker pool) and can choose the length of the queue; a longer queue implies a higher matching probability for the firm and a lower matching probability for the workers, and workers decide which queue to join by evaluating both the wage and the matching probability. The fact that the queue is dedicated to a firm instead of shared by all firms leads to important differences for both workers and firms. As stated above, workers in the directed search model face a trade-off between higher pay and a lower matching probability. In contrast, in our model, there is a shared queue for all jobs, and workers do not need to decide which queue to join; instead, workers make a trade-off between higher pay and a longer waiting time. On the firm side, under the directed search model, there is a trade-off between a lower profit margin and a higher probability of hiring, which has some similarities with our model; however, there is one key difference: In the directed search

model, the increase in the pay of one firm goes to increase its own matching probability, and there does not exist a notion of the common good. In contrast, the increase in the pay of a firm in our model increases the worker supply for the whole market. The shared worker pool as a common good is the key feature in our model but is outside the scope of the directed search model.

Workers' multi-apping behavior is a unique type of multi-homing. Therefore, the literature on multi-homing and platform competition is highly relevant. Regarding multi-homing, Armstrong 2006 builds models that allow one side of the market to be multi-homing and derive the equilibrium prices. The main finding is that the single-homing side of the platform is particularly valuable since it is the niche of the platform; therefore, platforms have the incentive to compete for the single-homing side, which involves charging a high price to the multi-homing side and passing a large proportion of the earnings to the single-homing side. Rochet and Tirole 2003 examine a setting largely motivated by the credit card industry, assuming that a platform charges a transaction fee to the two sides of the platform. They find that in equilibrium, all retailers accept all credit cards (i.e. multi-homing), but consumers only choose their preferred card (i.e. single-homing). More recently, Allon, D. Chen, and Moon 2023 empirically examine strategic workers' multi-homing behavior using a dataset that includes detailed trip-level data on where drivers work and which platform they choose. The study finds that about 56% have accepted jobs on multiple platforms within the observed timeframe. The analysis also shows that multi-homing can be a burdensome and costly decision for drivers even though it improves driver earnings. Notably, the study suggests that encouraging multi-homing improves system efficiency, resulting in higher hourly earnings for workers and enhanced service levels. Relatedly, Chitla et al. 2023 empirically examines ride-hailing customers' multi-homing behavior and finds that only 16.6% of customers use both Uber and Lyft within the observed timeframe. These findings suggest a potential asymmetry in the two-sided market, as the supply side appears more likely to multi-home than the customer side, aligning with the observations in Rochet and Tirole 2003. Furthermore, a number of recent papers have examined the implication of multi-homing on firms' profits and social welfare and yielded different conclusions. Bryan and Gans 2019 analyze the competition among ride-hailing platforms when firms compete on both the price and the passenger wait time. The authors introduce an adapted Hotelling model to capture the service differentiation of the wait time and analyze various market structures regarding the competition level and the multi-homing capability of the two sides. The key finding is that multi-homing has a complicated impact on platform competition, and restricting drivers' multi-homing may reduce social welfare. Bernstein, DeCroix, and Keskin 2021 analyze the competition between platforms in the sharing economy when both sides of the market care about congestion. The authors compare the two settings of drivers single-homing and multi-homing and conclude that driver multi-homing is bad for all players but may be attractive to individual drivers. Loginova, Wang, and Liu 2019 analyze multi-homing in the context of ride-hailing and show that multi-homing on either

side improves social welfare. Amaniamedani, Aouad, and Freund 2023 study a duopoly competition in spatial matching markets where suppliers multihome, using a stochastic game-theoretic model. Interestingly, they also identify an asymmetry between the larger and the smaller platform in matching: the larger platform may be incentivized to take more responsibility for maintaining the density of supply, which is conceptually related to the free-riding results in our model.

In addition, Siddiq and Taylor 2022, L. Chen et al. 2020, and Tan and Zhou 2020 also study two-sided platform competition in the gig economy, although the role of multihoming is not the focus. Ahmadinejad et al. 2019 study competition between two ride-hailing platforms. Interestingly, they also view available workers as a shared resource and show that competition can lead to market failure as a “tragedy of the commons” type outcome. However, since competition is the central topic of this stream of literature, it focuses primarily on how two-sided competition among platforms providing similar services affects surplus and prices. In our paper, we study this setting as a special case of our model (Section 4.1), reaching similar conclusions. But our analysis is more general in the sense that we also study an alternative setting where loyal customers are allowed, and show that market collapse will never happen in equilibrium as long as one of the firms has sufficient loyal customers.

## 2 Model setup

As previewed in the introduction, we model the decisions of the firms and the workers as a multistage game, the order of which is illustrated in Fig. 1. In our analysis, we use bold letters ( $\mathbf{v}$  for prices,  $\mathbf{p}$  for worker pay, and  $\boldsymbol{\mu}$  for job arrival rates) to denote vectors, which are equivalent to listing each firm’s decision individually as  $(v_1, v_2, \dots, v_N)$ , etc.

We solve the game using backward induction. Thus, we present the model in reverse order. We start with the workers’ decisions, starting with the acceptance decision (Section 2.1.1), followed by the arrival rate of the workers (Section 2.1.2). Next, we present the firms’ decisions, including pay setting (Section 2.2.1), demand realization (Section 2.2.2), and finally the pricing decision (Section 2.2.3). In Section 2.3, we introduce the equilibrium concept. In Section 2.4, we formalize the definition of free-riding.

### 2.1 Workers’ decisions

As shown in Fig. 1, we assume that workers act after the firms have made their decisions. So at this stage, worker pay ( $\mathbf{p}$ ), prices ( $\mathbf{v}$ ), and the arrival rate of the jobs ( $\boldsymbol{\mu}$ ) are determined and therefore treated by the workers as given. We assume a shared worker pool, where participating workers do not commit to any single firm but instead receive job offers from all firms (that is, “multiapping”). Workers may reject jobs that do not provide sufficient pay and make these decisions considering all potential jobs (current and future) in the

market. Workers have preferences for jobs, not firms; that is, they evaluate each job only in terms of its pay rate and expected duration.<sup>6</sup>

In Section 2.1.1 we analyze how workers decide which jobs to accept, conditional on participating in the gig economy; Section 2.1.2 examines their decision to enter the market in the first place.

### 2.1.1 An individual worker's decision

We assume a simple but plausible model of worker behavior. The number of workers is assumed to be large, and hence their behavior can be modeled as a rational expectation equilibrium. (See Jordan and Radner 1982 and Kalai 2004.) In this equilibrium, each worker expects to receive a sequence of arriving job offers according to a stationary stochastic process, where each (random) arriving job  $\omega$  has an associated pay rate  $p_\omega$  and takes time  $t_\omega$  to complete. When offered a job, workers can decide to accept or reject it. If they accept the job  $\omega$ , they collect the reward  $p_\omega t_\omega$  and are busy for  $t_\omega$  units of time; if they reject it, they must wait for the next offer. For a given worker policy  $\pi$ , let  $R_\pi(t)$  denote the (random) reward process (earnings) the policy  $\pi$  generates for the worker.

We assume that workers are risk neutral and the only information they have about the market is an estimate of their optimal expected discounted earnings conditioned on being in the idle (available) state, defined as

$$V_\delta = \sup_{\pi} \left\{ E \left[ \int_0^\infty e^{-\delta t} R_\pi(t) dt \right] \right\}$$

(We index this value by  $\delta$  because we will look at the limiting case  $\delta \rightarrow 0$ .) Workers' expectations are rational in the sense that, in equilibrium, their discounted earnings are indeed  $V_\delta$ . For example, this expectation could be based on previous earnings under similar market conditions. Given a job offer  $(p_\omega, t_\omega)$ , a worker's optimal decision is then to accept the job if and only if

$$p_\omega t_\omega + e^{-\delta t_\omega} V_\delta \geq V_\delta$$

This is true because the left-hand side above is the expected discounted earnings from accepting the job (i.e., the reward received from the job,  $p_\omega t_\omega$ , plus the discounted value of being in the idle state  $t_\omega$  time units from now) and the right-hand side is the expected discounted earnings from rejecting the job and remaining in the idle state. Rearranging, we get  $p_\omega t_\omega \geq V_\delta(1 - e^{-\delta t_\omega})$ . For  $\delta$  tending to zero,  $e^{-\delta t_\omega} \rightarrow 1 - \delta t_\omega$  so this becomes  $p_\omega \geq \bar{V}$  where we assume that  $\bar{V} = \lim_{\delta \rightarrow 0} \delta V_\delta$  exists and is interpreted as the optimal expected earnings rate. In summary, we have the following proposition.

**Proposition 1.** *Suppose workers have rational expectations about their optimal expected earnings rate,  $\bar{V}$ . Then their optimal strategy is to accept all jobs  $\omega$  for which  $p_\omega \geq \bar{V}$ .*

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<sup>6</sup>This rules out loyalty, which we study in Section 5, or volume-based incentives to lock in workers.

An alternative microfoundation model for this strategy is provided in Appendix A.1, where, rather than assuming workers have rational expectations about their expected earnings per unit time, we assume workers have rational expectations about the arrival rates and pay rates of all  $N$  firms in the market, and they solve an average-reward MDP to determine an optimal strategy. The resulting strategy is the same, pointing to the robustness of Proposition 1.

### 2.1.2 Workers' participation decision and the equilibrium pool size

Given that workers behave according to the strategy in Proposition 1, we next analyze the resulting worker pool equilibrium. We assume that all workers have a common reservation wage  $w_0$  (the value per unit time of their outside option) and that there is an infinite supply of potential workers, which implies that the labor market is perfectly elastic. For workers who participate and follow the optimal strategy, let  $P$  denote their average earnings per job,  $T$  denote the average job duration and  $W$  denote their average waiting time to receive a job offer. Then in equilibrium, we must have:

$$P = w_0(W + T) \tag{1}$$

This follows since if  $P > w_0(W + T)$ , all outside workers strictly benefit by joining the gig economy, since their expected pay per job strictly exceeds the opportunity cost of their time waiting for a job plus their time spent doing the job (the right-hand side above). This surplus gain would trigger an infinite number of outside workers to participate, which in turn would make the waiting time  $W$  arbitrarily large, contradicting the fact that  $P > w_0(W + T)$ . Conversely, if  $P < w_0(W + T)$ , all workers would choose to exit the gig economy since their expected pay per job would be strictly less than the opportunity cost of their time participating. Hence, the only stable outcome with workers participating is Eq. (1). Note also that this condition implies that the average earnings per unit of time of workers in the market is  $w_0$ . This gives us the following corollary to Proposition 1:

**Corollary 1.** *When worker entry and exit are in equilibrium, Eq. (1) holds and the average earnings per unit time must be  $\bar{V} = w_0$ . Therefore, the worker's optimal strategy is to accept all jobs  $\omega$  paying  $p_\omega \geq w_0$ .*

Also, note that from Eq. (1), workers are indifferent between participating and not participating in the gig economy. Hence, without loss of generality, we can assume that each participating worker enters, waits for the first acceptable job offer (one paying  $w_0$  or greater), performs that job, and then exits (at which point they are free to reenter).

Let  $\lambda$  denote the number of workers per unit time entering the market (the worker *participation rate*). Under this interpretation, the gig economy operates like a queueing system in which workers “arrive” at rate  $\lambda$ , wait an amount of time  $W$  and are “served” by being offered an acceptable job. Combining this queueing interpretation and Proposition 1, we

can then write Eq. (1) as

$$\sum_{i, p_i \geq w_0} \frac{\mu_i}{\mu_{\geq w_0}} p_i t_i = w_0 \left( \sum_{i, p_i \geq w_0} \frac{\mu_i}{\mu_{\geq w_0}} t_i + W(\lambda; \mu_{\geq w_0}) \right) \quad (2)$$

where  $\mu_{\geq w_0} = \sum_{i, p_i \geq w_0} \mu_i$  is the demand rate of jobs paying  $w_0$  or higher (acceptable jobs) and  $W(\lambda; \mu_{\geq w_0})$  is the worker's expected waiting time for an acceptable job, which is a function of the worker participation rate  $\lambda$  and the acceptable job demand rate  $\mu_{\geq w_0}$ .

Lastly, we assume that the waiting time function is the M/M/1 formula:

$$W(\lambda; \mu_{\geq w_0}) = \frac{1}{\mu_{\geq w_0} - \lambda}, \quad \mu_{\geq w_0} > \lambda \quad (3)$$

This formula holds without any assumption about the order of service; however, it does require that the demand rate of jobs is Poisson and that the participation rate of workers is Poisson. It is exact under these two assumptions. Combining (2) and (3) yields the following closed-form expression for the worker participation rate  $\lambda$  as a function of the pay rate  $\mathbf{p}$  and demand  $\boldsymbol{\mu}$ :

$$\lambda(\mathbf{p}; \boldsymbol{\mu}) = \left( \mu_{\geq w_0} - \frac{w_0}{\sum_{i, p_i \geq w_0} \frac{\mu_i}{\mu_{\geq w_0}} t_i (p_i - w_0)} \right)^+ \quad (4)$$

## 2.2 Other Decisions

Continuing to present the model in reverse order according to the sequence in Fig. 1, we consider the firms' decisions on worker pay and treating prices and demand rates as given. We then examine how prices and demand rates are set given the anticipated worker pay equilibrium.

### 2.2.1 Firms' pay decisions

Given the prices  $\mathbf{v}$ , demand rates  $\boldsymbol{\mu}$  and all other firms' pay  $\mathbf{p}_{-i}$ , firm  $i$  maximizes its profit by choosing  $p_i$ , anticipating its impact on worker decisions. As discussed in Section 2.1.1, if  $p_i < w_0$ , no worker will accept a job from firm  $i$ , resulting in zero profit; else, firm  $i$ 's profit can be expressed as:

$$\pi_i(p_i, \mathbf{p}_{-i}; \mathbf{v}, \boldsymbol{\mu}) = (v_i - p_i) t_i \cdot \frac{\lambda(p_i, \mathbf{p}_{-i}; \boldsymbol{\mu})}{\mu_{\geq w_0}} \cdot \mu_i \quad (5)$$

where  $\pi_i(p_i, \mathbf{p}_{-i}; \mathbf{v}, \boldsymbol{\mu})$  denote firm  $i$ 's expected profit rate. This profit rate consists of three terms: (1)  $(v_i - p_i) t_i$ , the profit per job of firm  $i$ ; (2)  $\lambda(p_i, \mathbf{p}_{-i}; \boldsymbol{\mu}) / \mu_{\geq w_0}$ , the probability that an arriving job is served (the *completion rate*), which is the same for all firms that

pay at least  $w_0$ ; and (3)  $\mu_i$ , the arrival rate of jobs for firm  $i$ . From these components, we see that firm  $i$  faces a trade-off: if the worker pay  $p_i$  is too high, it reduces the profit per job  $(v_i - p_i)t_i$ ; however, if  $p_i$  is too low, it lowers the job completion rate due to a reduction in the worker arrival rate  $\lambda(p_i, \mathbf{p}_{-i}; \boldsymbol{\mu})$ , since from (4) the arrival rate strictly increases in any firm's pay rate  $p_i$ . Firm  $i$  then solves the problem below to choose its pay:

$$\max_{p_i \geq w_0} \pi_i(p_i, \mathbf{p}_{-i}; \mathbf{v}, \boldsymbol{\mu}) \quad (6)$$

If the maximum profit in Eq. (6) is negative, firm  $i$  is better off not participating in the gig economy.

The formulation in Eq. (6) highlights the complex interactions between the pay decisions of firms: all firms benefit from a larger pool of workers (and hence a higher  $\lambda$ ), to which they each contribute through Eq. (4). However, if other firms pay sufficiently to maintain a high  $\lambda$ , each individual firm has an incentive to lower its pay rate to increase its profit per job served.

The outcome of all  $N$  firms' decisions is the equilibrium pay rate vector  $\mathbf{p}^*$ , which depends on both the prices and demand rates. We denote the equilibrium pay rates by  $\mathbf{p}^*(\mathbf{v}, \boldsymbol{\mu})$  and the corresponding equilibrium profit rates by  $\boldsymbol{\pi}^*(\mathbf{v}, \boldsymbol{\mu})$ . The pay rate of each firm  $p_i^*$  is the best response given the pay rates of other firms, ensuring that no firm can unilaterally improve its profit by deviating from  $p_i^*$ . That is,  $\mathbf{p}^*$  satisfies

$$p_i^* = \arg \max_{p_i} \pi_i(p_i, \mathbf{p}_{-i}^*; \mathbf{v}, \boldsymbol{\mu}), \quad \forall i \quad (7)$$

If  $p_i^* < w_0$ , it implies that firm  $i$  cannot earn a positive profit given  $\mathbf{p}_{-i}^*$ ,  $\mathbf{v}$ , and  $\boldsymbol{\mu}$ , and thus choose not to participate in the gig economy.

### 2.2.2 Demand realization

In this stage, demand is realized given the prices  $\mathbf{v}$ . We do not assume any specific demand behavior other than the order of events. That is, the demand behavior may depend on the fixed prices  $\mathbf{v}$  and also anticipate the lower stage equilibria (e.g., the equilibrium pay, the worker arrival rate). For example, demand may be not only sensitive to prices but also service quality metrics that depend on the number of workers available in equilibrium. Section 4 will study how different demand behaviors (e.g., competitive or non-competitive) influence our results. We denote the resulting equilibrium demand rate by  $\boldsymbol{\mu}^*(\mathbf{v})$ <sup>7</sup>, which reflects these decisions at the aggregate level.<sup>8</sup>

<sup>7</sup>Given  $\mathbf{v}$ , the equilibrium demand rate  $\boldsymbol{\mu}^*(\mathbf{v})$  can be determined by solving a fixed-point problem: find  $\boldsymbol{\mu}$  such that  $\boldsymbol{\mu} = D(\mathbf{v}, \mathbf{p}^*(\mathbf{v}, \boldsymbol{\mu}), \lambda(\mathbf{p}^*(\mathbf{v}, \boldsymbol{\mu}), \boldsymbol{\mu}))$ , where  $D$  represents a demand function that depends on price, pay, and the worker arrival rate.

<sup>8</sup>In the simulation in Section 5, we provide an explicit customer model as players in the game who make strategic decisions about demand.

### 2.2.3 Firms' pricing decisions

In the first stage, each firm selects its price, anticipating its impact on the profit margin, demand rate, and the resulting effect on the job completion rate, which depends on the worker pool. More formally, given the prices of all other firms, denoted by  $\mathbf{v}_{-i}$ , firm  $i$  solves the following optimization problem to determine its price  $v_i$ :

$$\max_{v_i} \pi_i^*(v_i, \mathbf{v}_{-i}, \boldsymbol{\mu}^*(v_i, \mathbf{v}_{-i})) \quad (8)$$

Recall that  $\pi_i^*(\cdot)$  is the profit rate for firm  $i$  under the worker pay equilibrium, as defined in Section 2.2.1. This profit rate is a function of the prices  $\mathbf{v}$  and the equilibrium demand rates  $\boldsymbol{\mu}^*(\mathbf{v})$ , as established in Section 2.2.2. The outcome of the pricing decisions by all  $N$  firms constitutes the equilibrium prices  $\mathbf{v}^*$ , defined by

$$v_i^* = \arg \max_{v_i} \pi_i^*(v_i, \mathbf{v}_{-i}^*, \boldsymbol{\mu}^*(v_i, \mathbf{v}_{-i}^*)), \quad \forall i \quad (9)$$

## 2.3 Summary of equilibrium

To summarize, the equilibrium concept for this game is a multi-stage Nash equilibrium. In each stage, players reach a Nash Equilibrium, accounting for both the outcomes of previous stages and the anticipated outcomes of future stages. The equilibrium outcome is represented by a tuple  $(\mathbf{v}^*, \boldsymbol{\mu}^*, \mathbf{p}^*, \lambda^*)$ , where each component denotes the equilibrium values at each stage:  $\mathbf{v}^*$  for prices,  $\boldsymbol{\mu}^*$  for demand,  $\mathbf{p}^*$  for pay rates, and  $\lambda^*$  for worker participation. We determine an equilibrium using backward induction and solve the first four stages of the game in reverse order.<sup>9</sup>

Note that in our setup, firms make simultaneous decisions in both the pricing and pay-setting stages, with each stage reaching a Nash equilibrium. This structure implicitly assumes that demand (customer arrivals) adjusts more slowly than supply (worker arrivals). In addition, though demand realization is modeled as a stage for analysis, it is not a true decision-making stage, as customers are not modeled as active players. In Section 5, we validate the robustness of our results by exploring alternative sequences of events and explicitly modeling customers as strategic players through numerical analysis.

## 2.4 Firm participation and free-riding

Central to our model of a gig economy is the existence of a labor pool that is shared by all firms with workers who are free to enter and exit this pool. As shown in Section 2.1, it is optimal for workers to accept any job that pays at least  $w_0$ , which implies that any

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<sup>9</sup>While the worker's acceptance decision (the last stage in Fig. 1) is also a stage, Section 2.1.1 demonstrates that the optimal worker strategy is a constant threshold policy, which remains the same across all settings. As a result, this decision is not included in the equilibrium output tuple  $(\mathbf{v}^*, \boldsymbol{\mu}^*, \mathbf{p}^*, \lambda^*)$ .

given firm only needs to pay  $w_0$  to have all its jobs accepted by workers. This leads to the following definition of firm participation.

**Definition 1** (Firm Participation). *A firm  $i$  is said to be participating in the gig economy if and only if  $p_i \geq w_0$  and  $\mu_i > 0$ .*

However, note that (3) and (4) imply that to have a strictly positive completion rate, there must be a strictly positive queue of idle workers. To support these idle workers, some firms must pay strictly more than  $w_0$ , otherwise workers earn strictly less than  $w_0$  and would not participate. This suggests the possibility of free-riding. Formally:

**Definition 2** (Free-Riding). *Consider two participating firms,  $i$  and  $j$ . Then we say that firm  $i$  is free-riding on firm  $j$  if  $p_i = w_0 < p_j$ .*

In short, the queue of idle workers is a public good that benefits all firms by enabling a strictly positive completion rate. However, firms paying  $w_0$  do not contribute to maintaining this queue; they are free-riding firms that pay strictly more than  $w_0$ . Our main results in Section 3 and Section 4 establish when free-riding occurs in equilibrium, the magnitude of the relative pay rates of participating firms, and the conditions under which such an equilibrium exists.

### 3 Main results

In this section, we analyze equilibrium outcomes  $(\mathbf{v}^*, \boldsymbol{\mu}^*, \mathbf{p}^*, \lambda^*)$  of the multistage game defined in Section 2. For brevity, we sometimes omit the word “equilibrium” and refer to equilibrium values  $\mathbf{v}^*$ ,  $\boldsymbol{\mu}^*$ ,  $\mathbf{p}^*$ ,  $\lambda^*$  directly as prices, demand rates, worker pay and worker arrival rates. Since the duration of the jobs,  $t_i$ , is assumed to be exogenous, in the rest of the paper, we normalize it to one for all firms, i.e.  $t_i = 1$ ,  $i = 1, \dots, N$ , to simplify the presentation of the results.<sup>10</sup>

In Section 3.1, we first present a result on worker and firm participation (Section 3.1.1), offering structural insights into equilibria with positive worker participation ( $\lambda^* > 0$ ). We then characterize the properties of these equilibria, highlighting the free-riding effect (Section 3.1.2). Finally, in Section 3.2, we present results on the existence and uniqueness of possible equilibria.

#### 3.1 Participation and free-riding

##### 3.1.1 Firm and worker participation

Our first result shows that worker and firm participation are inextricably linked.

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<sup>10</sup>Our proofs in the appendix allow heterogeneous  $t_i$ .

**Proposition 2** (Firm Participation). *For any equilibrium in which workers participate ( $\lambda^* > 0$ ), it must be true that all firms with  $v_i^* > w_0$  participate and are profitable.*

In other words, in equilibrium, it is not possible for some firms to participate while others with  $v_i^* > w_0$  choose not to enter. This result is intuitive; given a shared worker pool with positive worker participation, any non-participating firm with  $v_i^* > w_0$  can always deviate and improve its profit by paying  $w_0$  and participating. We say that an equilibrium with participation results in *market formation*.

### 3.1.2 Characteristics of equilibria with $\lambda^* > 0$

To characterize equilibria, we define the following quantity for each firm  $i$ , given the price and demand rate  $(v_i, \mu_i)$ :

$$s_i \triangleq (v_i - w_0)\mu_i \quad (10)$$

Note the first term,  $(v_i - w_0)$ , is the maximum profit margin that firm  $i$  can generate per job given price  $v_i$  (assuming it pays workers the minimum acceptable rate,  $w_0$ ). The second term is the demand rate, which is also the expected maximum number of job hours the firm can fulfill per calendar hour (assuming there is always a worker available for each job). Thus,  $s_i$  represents the maximum potential profit rate that firm  $i$  can achieve under price  $v_i$  and demand  $\mu_i$ .

Given an equilibrium  $(\mathbf{v}^*, \boldsymbol{\mu}^*, \mathbf{p}^*, \lambda^*)$ , we then define the *size* of firm  $i$  as

$$s_i^* = (v_i^* - w_0)\mu_i^* , \quad (11)$$

and for two firms  $i$  and  $j$ , we say that firm  $i$  is larger than firm  $j$  if  $s_i^* > s_j^*$ . Size  $s_i^*$  can also be viewed as a measure of the *surplus* that firm  $i$  brings to the market in equilibrium. As reflected by Eq. (11), the size of a firm is a demand-side decision, but since pricing and demand realization are the first two stages in the multistage game, this decision is made anticipating the supply-side equilibrium. Our main theorem for equilibria with market formation is:

**Theorem 1.** *Consider a gig economy with  $N$  firms. Then for any equilibrium  $(\mathbf{v}^*, \boldsymbol{\mu}^*, \mathbf{p}^*, \lambda^*)$  with  $\lambda^* > 0$ , there exists a threshold size  $s_\sigma$  and common profit rate  $\bar{\pi}$  such that each participating firm falls into one of the two categories:*

1.  $s_i^* \leq s_\sigma$  (small firms): *The firm pays  $p_i^* = w_0$  and free-rides on the large firms.*
2.  $s_i^* > s_\sigma$  (large firms): *The firm pays  $p_i^* > w_0$  such that  $\pi_i^* = \bar{\pi}$  regardless of  $s_i^*$ .*

*Moreover, there must be at least one large firm.*

In other words, each participating firm either pays the minimum rate  $w_0$ , or pays strictly more than  $w_0$  and achieves the same hourly profit rate. More precisely, note (5) shows that the hourly profit of firm  $i$  is given by  $\pi_i = (v_i - p_i) \frac{\lambda}{\mu_{\geq w_0}} \mu_i$  (assuming normalized job

duration  $t_i = 1$ ). Therefore, given two participating firms  $i, j$  such that with  $s_i^*, s_j^* > s_\delta$ , we must have  $(v_i^* - p_i^*)\mu_i^* = (v_j^* - p_j^*)\mu_j^*$  in equilibrium. Thus, the equilibrium worker pay  $p_i^*$  causes the profit margin of the firm  $i$ ,  $(v_i^* - p_i^*)$ , to be inversely proportional to its demand  $\mu_i^*$ . This means that firms with more demand must have proportionately lower profit margins, which is a striking diseconomy of scale. Small firms ( $s_i^* \leq s_\sigma$ ), in contrast, pay the minimum rate  $p_i^* = w_0$  and therefore are free-riding on the larger firms.

The next natural question is: Under what conditions does such an equilibrium exist? And is it unique? These are difficult questions to answer: as we did not impose any conditions on our demand model, some choices may lead to a unique equilibrium, many equilibria, or no equilibrium at all. We will study the impact of various demand models in Section 4. However, given pricing and demand decisions, the supply-side decisions are much more well-behaved. We will see now in Section 3.2 that, given any outcome  $(v, \mu)$  of the two demand-side stages, we can characterize the existence and uniqueness of the supply-side sub-game equilibria  $\mathbf{p}^*, \lambda^*$ .

### 3.2 Existence and uniqueness of the worker pay equilibrium

Our next main theorem shows that the existence and uniqueness of the worker pay subgame equilibrium hinge on the individual and total sizes of the firms:

**Theorem 2.** *Consider  $N$  firms. Then for any values in the first two stages,  $(v_i, \mu_i)$ ,  $i = 1, \dots, N$ , the following holds for the worker pay subgame:*

1. *A Nash equilibrium with  $\lambda^* = 0$  exists if and only if none of the firms is large enough to create a market on its own:*

$$s_i = (v_i - w_0)\mu_i \leq w_0, \quad \forall i. \quad (12)$$

2. *A Nash equilibrium with  $\lambda^* > 0$  exists if and only if the total size of the firms with  $v_i > w_0$  satisfies*

$$\sum_{i=1}^N s_i > w_0. \quad (13)$$

*Moreover, this equilibrium is unique and satisfies the properties described in Theorem 1.*

3. *Combining (a) and (b), market formation (an equilibrium with  $\lambda^* > 0$ ) is the unique equilibrium if and only if the largest firm can create a market on its own:*

$$\max_i \{s_i\} > w_0. \quad (14)$$

Theorem 2 provides several important insights. In particular, suppose that firm  $i$  with  $v_i > w_0$  is not large enough to sustain a market on its own (i.e.,  $0 < s_i \leq w_0$ ). Then Item 2

implies that firm  $i$  can still operate profitably as long as the combined size of all other firms is large enough (i.e.,  $\sum_{j \neq i, v_j > w_0} s_j > w_0 - s_i$ ). In other words, there is an aggregation effect in which a sufficient number of small firms that cannot operate independently can create a viable market.

However, such an equilibrium may not be easily achieved, depending on the size of the largest firm. For example, assume the total size of firms with  $v_i > w_0$  is above the critical threshold (i.e.,  $\sum_{i, v_i > w_0} s_i > w_0$ ), but each individual firm is small (i.e.,  $s_i \leq w_0$  for all  $i$ ). In this case, no one firm is large enough to operate independently, but through coordinated entry, all firms with  $v_i > w_0$  could collectively sustain the market. But this requires coordinated entry decisions, which can be challenging to achieve. The alternative outcome is that none of the firms participates. Item 1 shows that if all firms are too small to support a gig economy individually, they may collectively fail to create one, even when coordinated entry is theoretically possible. Hence, when  $s_i \leq w_0$  for all  $i$  but  $\sum_{i, v_i > w_0} s_i > w_0$ , an equilibrium with  $\lambda^* = 0$  (no market formation) can still occur.

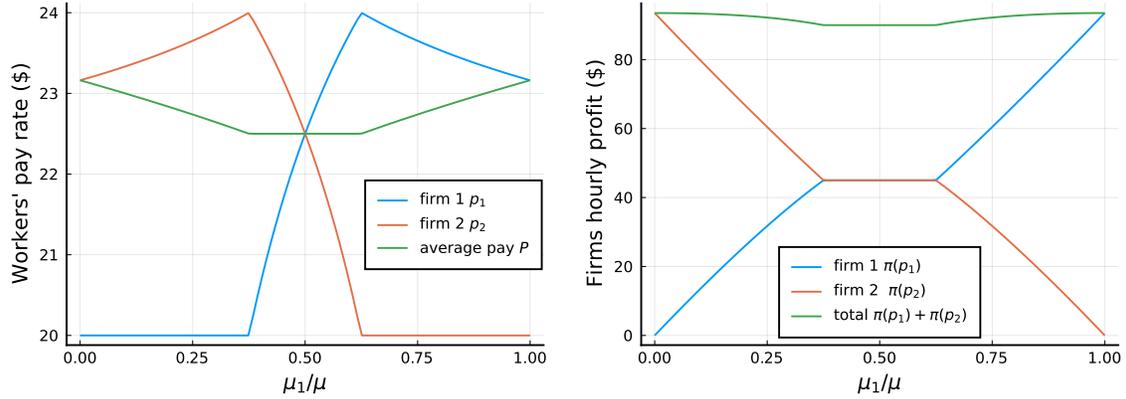
In contrast, Item 3 shows if a single firm is large enough to “jumpstart” the gig economy on its own (i.e.,  $\max_i \{s_i\} > w_0$ ), then market formation ( $\lambda^* > 0$ ) is the unique equilibrium and Theorem 1 always applies. This highlights the critical role that the existence of a large firm plays in forming and stabilizing the gig economy, an outcome that resonates strongly with the historical role that Uber played in accelerating the gig economy.

### 3.3 A two-firm example

To give a numerical illustration of the supply-side sub-equilibria outcomes. We fix the pricing decisions and consider two firms that earn the same revenue per job ( $v_1 = v_2$ ). We then vary the second-stage demand arrival rates while keeping the total demand rate constant:  $\mu_1 + \mu_2 = \mu$ , which is equivalent to varying their sizes while keeping the total size ( $s_1 + s_2$ ) constant. For each choice, Figure 2 shows the corresponding supply-side sub-equilibrium.

Looking at the equilibrium pay rates in Figure 2a, we observe that the smaller firm can capture up to 38% of the total job arrival rate while still free-riding by paying only  $w_0$ . In the most extreme scenario, the profit margin of the smaller firm reaches \$10 per job, while the larger firm’s margin is only \$6. Figure 2b shows the hourly profits of both firms, illustrating Result 2 in Theorem 1: if the smaller firm pays more than  $w_0$ , both firms earn exactly the same hourly profit.

The total profit line in Figure 2b also reveals that the equilibrium produces slightly lower total profit than a monopoly with the same total demand (represented by the endpoints of the graph), indicating that the smaller firm’s free-riding slightly reduces overall market profitability. This example demonstrates that free-riding has a substantial effect: even a firm that is only slightly smaller than another can gain a significant labor-cost advantage



(a) The equilibrium worker pay  $p_1$ ,  $p_2$  and the average  $P = (\mu_1 p_1 + \mu_2 p_2)/\mu$  at the Nash equilibrium. (b) Firms' hourly profits  $\pi(p_1)$ ,  $\pi(p_2)$ , and total profit  $\pi(p_1) + \pi(p_2)$  at the Nash equilibrium.

Figure 2: Equilibria in a setting with two firms where the total size of the two firms is constant.

Note:  $\mu_1 + \mu_2 = 20/\text{hour}$ ,  $w_0 = 20\$/\text{hour}$ ,  $t_1 = t_2 = 1$  hour,  $v_1 = v_2 = 30\$/\text{hour}$ . We vary the relative size of the two firms, from  $\mu_1/\mu = 0$  (firm 2 is a monopoly) to  $\mu_1/\mu = 1$  (firm 1 is a monopoly).

in equilibrium.

## 4 Demand competition models and equilibrium existence

In this section, we explore how the demand side influences the existence of the equilibria characterized in Theorems 1 and 2. We show that competition significantly impacts whether a market can form. In particular, in Section 4.1, we show that under perfect demand competition the only equilibrium for the worker pay subgame (and hence the multistage game) results in market collapse ( $\lambda^* = 0$ ). However, Section 4.2 shows that if at least one firm has a sufficient number of loyal customers to sustain the market on its own, market collapse is never an equilibrium. Furthermore, all other firms with a non-zero number of loyal customers will also participate. Section 4.3 then investigates a special case of Section 4.2, where all customers are loyal to specific firms and each firm's demand depends solely on its own price, i.e., the case of no demand competition. We show that a price equilibrium involves each firm choosing a price to maximize its own size. That is, despite the labor-cost advantages of being small, a firm prefers to become as large as possible.

Combining these results implies that free-riding is robust given any demand-side decisions that induce market formation, but whether an equilibrium with market formation exists depends critically on demand-side competition. At least one firm must exhibit some degree of market power, and if so, market collapse is never an equilibrium, and the results in

Theorem 1 apply.

## 4.1 Perfect competition

Consider the setting of perfect price competition. That is, there is a fixed demand rate for the entire market, and whichever firm offers the lowest price captures all of the demand (infinite own-price elasticity). We then have the following proposition:

**Proposition 3.** *Consider two firms under perfect price competition. Then the only equilibrium is market collapse ( $\lambda^* = 0$ ).*

This result extends easily to  $N$  firms. The intuition is straightforward: in a perfectly competitive environment, if the competitor’s price is higher than  $w_0$ , then a firm strictly benefits from setting its price to be  $\epsilon$  lower than its competitor to capture the entire market. This leads to a race to the bottom, in which prices are driven down to  $w_0$ , at which point no firm achieves a positive size - essentially the classic outcome of Bertrand competition. Proposition 3 shows that when demand-side competition is intense, labor-side free-riding is irrelevant because the demand-side equilibrium itself precludes market formation.

## 4.2 Imperfect competition

Of course, real-world competition is rarely as intense as perfect competition. For example, companies such as Uber and Lyft use loyalty programs and other incentives to encourage repeated usage from customers (Skift 2023). These initiatives effectively reduce price sensitivity, making loyal customers less likely to switch to competing platforms even with small price differences. Empirical evidence supports this view: Chitla et al. 2023 finds that only 16.6% of ride-hailing customers use both Uber and Lyft, and among these customers, they consider both platforms just 43.4% of the time. The existence of dedicated demand can help firms maintain some market power and reduce the intensity of price cutting behavior.

Indeed, our next result, Proposition 4, demonstrates that as long as at least one firm can achieve a size strictly higher than  $w_0$ , regardless of other firms’ pricing strategies, then market collapse ( $\lambda^* = 0$ ) is never an equilibrium outcome.

**Proposition 4.** *Consider  $N$  firms. Suppose there exists a firm  $i$  and a price  $v_i$  satisfying*

$$s_i = (v_i - w_0)\mu_i^*(v_i; \mathbf{v}_{-i}) > w_0, \quad (15)$$

*for any pricing strategies  $\mathbf{v}_{-i}$  chosen by other firms. Then, any equilibrium must satisfy  $\lambda^* > 0$  (market formation). Moreover, if there exists another firm  $j \neq i$  and price  $v_j$ , such that  $s_j = (v_j - w_0)\mu_j(v_j; \mathbf{v}_{-j}) > 0$  for any pricing strategies  $\mathbf{v}_{-j}$  chosen by other firms, then firm  $j$  will also participate in the gig economy and earn a positive profit.*

Condition (15) can be interpreted as requiring a firm’s loyal customer base to generate sufficient demand. For instance, suppose firm  $i$  has a loyal customer segment,  $\mu_{i,l}$ , that

exclusively requests services from firm  $i$ , provided its price remains within a reasonable range (e.g.,  $v_i \leq \bar{v}_i$ ). Even if firm  $i$  engages in perfect price competition with other firms for the rest of the market, it can still achieve a size of  $(\bar{v}_i - w_0)\mu_{i,l}$  through its loyal customers.

Proposition 4 states that if the size generated by the loyal customer base exceeds  $w_0$ , i.e.,  $(\bar{v}_i - w_0)\mu_{i,l} > w_0$ , then firm  $i$  will participate in the gig economy, enabling a positive worker pool. Furthermore, any other firm with even a small amount of loyal demand will strictly benefit from participation as well. Consequently, market collapse ( $\lambda^* = 0$ ) cannot occur in equilibrium.

Admittedly, Proposition 4 does not guarantee the existence of an equilibrium with  $\lambda^* > 0$ . It remains possible that there are no equilibria and that the pricing stage of the game enters a cycle. For example, firms may continue to undercut each other due to perfect competition over the non-dedicated portion of demand. However, if all firms choose not to participate, the firm with  $s_i > w_0$  will strictly benefit from re-entering, leading to a cyclic dynamic. Such dynamics can be stabilized when the demand function takes a smoother form (e.g., the logit demand model), where undercutting does not result in abrupt changes in profit. While we do not theoretically study such demand models and cycles, the simulation approach from Section 5 will be able to compute equilibria and stable cycles. Nonetheless, as long as a market exists - even in the cyclic scenario described above - free-riding on pay will persist whenever a market forms.<sup>11</sup>

### 4.3 Independent demand (A special case of Section 4.2)

A special case of Section 4.2 is when each firm's demand depends solely on its own price, i.e., all customers are loyal customers. Although a special case, this scenario is practically relevant. For instance, it applies when platforms offer different services but share the same supply pool, such as Lyft (ride-hailing) and Doordash (food delivery).

In this setting, each firm  $i$  sets its prices  $v_i$ , which impacts its demand rate. But because there is no demand competition, a firm's demand only depends on its own price and not the prices of other firms. Consequently, the size of the firm  $i$ , i.e.,  $s_i^*$ , defined by Eq. (11), becomes a function of firm  $i$ 's own equilibrium price  $v_i^*$ . In particular, the *maximum size*  $\bar{s}_i$  of firm  $i$  is defined as:

$$\bar{s}_i \triangleq \max_{v_i} s_i, \text{ where } s_i = (v_i - w_0)\mu_i^*(v_i) \quad (16)$$

In this special case, we are able to explicitly identify a dominant strategy equilibrium for the pricing stage of the game among the firms:

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<sup>11</sup>As a reminder, our model accommodates complex demand functions that may depend on later stages of the game, such as the size of the worker pool which can be viewed as a proxy for service quality. In this sense, Proposition 4 holds even when service quality influences demand.

**Proposition 5.** *Suppose that firms’s demand depends only on their own price and that there are  $N$  firms with maximal sizes  $(\bar{s}_1, \bar{s}_2, \dots, \bar{s}_N)$ . If  $\sum_i \bar{s}_i > w_0$ , then there exists a dominant strategy equilibrium for the pricing stage of the game,  $(v_1^*, \dots, v_N^*)$ , where each firm chooses to maximize its size, that is,  $s_i^* = \bar{s}_i, \forall i$ .*

To illustrate, the numerical example in Appendix Appendix D demonstrates that in the non-competitive setting, the worker pay equilibrium ensures that any firm’s equilibrium profit is non-decreasing with respect to the sizes of all firms (including its own). Consequently, it becomes a dominant strategy for each firm to maximize its size whenever possible.

Combining this result with Theorem 1 and Theorem 2, we have a complete characterization of the equilibria in the non-competitive demand case. Proposition 5 characterize the first two stages (demand-side), while Theorem 1 and Theorem 2 characterize the last stages (supply-side). In particular, recall that by Theorem 2, there is an aggregation effect among firms in terms of market formation, and these results apply in the non-competitive case. Specifically, if a single firm is large enough to “jumpstart” the gig economy on its own (i.e.,  $\max_i \{\bar{s}_i\} > w_0$ ), a gig economy with free-riding as in Theorem 1 will form, and other firms will find it profitable to enter. This finding is consistent with the rapid growth of the gig economy in the US in the wake of Uber - a heavily funded, hypergrowth company. It suggests that Uber’s ride-hailing growth created a large pool of workers willing to operate as independent contractors, which then made it profitable for other gig businesses, such as food and package delivery, to form.

## 5 Robustness check through numerical analysis

Our model is intentionally parsimonious to make the equilibria analysis tractable, but this naturally raises questions about the overall robustness of our insights. In particular, we assumed perfect supply elasticity with an infinite worker pool, while, in reality, recruiting additional workers may require increasing pay rates. We also assumed perfectly flexible, multi-apping workers, when we know some workers are loyal to certain firms. In our model all firms share the same pool of workers, so there are no density effects that might increase the service quality (e.g., shorter pickup times in ridesharing) for large firms. Lastly, we modeled the game sequentially, with firms making demand-side decisions before supply-side decisions, while actual demand- and supply-side decisions may be made simultaneously.

To test the robustness of our results to these model assumptions, we built a numerical model to simulate price, pay, supply, and demand equilibria under competition between *two* firms. This simulation tool, which is publicly available<sup>12</sup>, can test much more general settings than our theory. We first describe the main features of the simulation, and then

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<sup>12</sup>The code is available at REDACTEDFORPEER-REVIEW

present results that illustrate the effect of relaxing our theory’s main assumptions.

## 5.1 Simulation setup

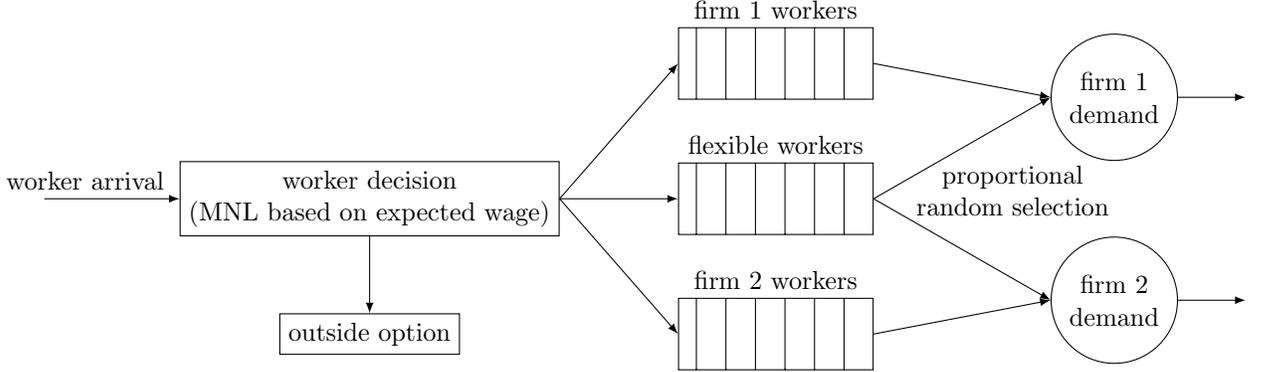


Figure 3: Queuing system with MNL Worker Decisions

*Note:* Workers arrive as a Poisson process. Arriving workers have four options: working for both firms (they join the “flexible” queue), being dedicated to firm 1 or 2 (they join a “dedicated queue” that can only serve one type of demand), or going with the outside option. The firms cannot differentiate between dedicated and flexible workers and assign a job uniformly randomly across all the compatible available workers. In equilibrium, the probability of a worker choosing each option must follow the multinomial logit choice model, where the utility of each option is its expected hourly wage. These wages are the expected pay of the worker divided by the expected time spent waiting and completing the task, given the chosen option. The outside option expected hourly wage is  $w_0$ .

**Worker decisions.** Our main theoretical results are derived under the assumption of infinite worker elasticity (workers accept any job paying above their outside option wage,  $w_0$ ). However, this simplification does not hold when we introduce more realistic supply curves.

To address this, we consider a more general worker decision model to represent which firms the worker chooses to work for. We assume that we have a pool of  $N_{\text{multiapp}}$  workers who have four alternatives. They can choose to be “flexible” and accept any job; they can choose to be “dedicated” and only accept jobs from one of the two firms (2 choices); or they can decline to participate and earn their baseline wage  $w_0$ . We assume that firms assign jobs uniformly at random across available flexible workers and their dedicated ones (e.g., it may be hard/illegal for firms to differentiate the two in practice). Formally, this system is modeled as three worker queues, two dedicated and one shared, as illustrated in Fig. 3.

The number of workers choosing each option follows a multinomial logit (MNL) model over the four outcomes, which allows us to introduce supply elasticity to wages. For example,

the average number of workers who chose the flexible option is:

$$N_{\text{multiapp}} \frac{\exp(\alpha_{\text{supply}} w_{\text{flex}})}{\exp(\alpha_{\text{supply}} w_{\text{flex}}) + \exp(\alpha_{\text{supply}} w_1) + \exp(\alpha_{\text{supply}} w_2) + \exp(\alpha_{\text{supply}} w_0)}, \quad (17)$$

where  $\alpha_{\text{supply}}$  is the worker sensitivity to wages, and  $w_{\text{flex}}$ ,  $w_1$ ,  $w_2$ , and  $w_0$  are the expected hourly wages of flexible or dedicated workers, respectively. Similar expressions hold for the other three options. Supply is “in equilibrium” when the number of workers of each type satisfies this MNL model. Finding such an equilibrium is non-trivial because the wage depends on the number of workers, and the number of workers depends on the wages.

This worker behavior model generalizes our theoretical model by accommodating smoother supply curves and more nuanced worker decision-making. Notably, as  $\alpha_{\text{supply}}$  and  $N_{\text{multiapp}}$  become large, the model converges to the theoretical case of infinite elasticity. We verified that, in this limit, simulated equilibria match our theoretical predictions, with average wages aligning to  $w_0$  and flexible workers accepting any job paying  $> w_0$ .

**Demand decisions.** We aim to represent a range of demand models where demand depends on both price and service quality. We use a utility model where a customer receives utility  $U_{\text{service}} - v$  when they receive service, and utility  $c_{\text{no-service}}$  when they do not (i.e., no compatible worker is available).  $U_{\text{service}}$  is the value of a service for the customer (assumed the same for both firms),  $v$  is the price charged by the firm, and  $U_{\text{no-service}}$  is the utility of a rejected customer. We model the service level, denoted  $SL_1$ ,  $SL_2$ , as the probability that the customer is served when they request service. A higher service level  $SL_i$  has analogous dynamics to a lower pickup time in ridesharing or a faster delivery time in food delivery. Indeed, the service level is an increasing function of the average number of workers available. While it is not equivalent to pick-up/delivery time, it still allows us to model that customers prefer firms with more workers.

Customers make decisions based on the expected utility they will receive according to an MNL model. Let  $U_1$  and  $U_2$  be the expected utility of a customer of Firm 1 and Firm 2, then  $i \in 1, 2$ , we have:

$$U_i = SL_i(U_{\text{service}} - v_i) + (1 - SL_i)U_{\text{no-service}}. \quad (18)$$

Our first demand model is a non-competitive model similar to the one of Appendix C.3. Each firm has its own demand pool, and we use an MNL model where the outside option has zero utility to represent the demand curve. We reach a *demand equilibrium* if the demand rates  $\mu_1$  and  $\mu_2$  satisfy:

$$\mu_i = D_{\text{dedi-}i} \frac{\exp(\alpha_{\text{demand}} U_i)}{1 + \exp(\alpha_{\text{demand}} U_i)}, \quad (19)$$

where  $D_{\text{dedi-}i}$  is the maximum possible demand rate for Firm  $i$  and  $\alpha_{\text{demand}}$  is the demand sensitivity to utility. A high  $\alpha_{\text{demand}}$  means that we get all the demand if prices are

slightly lower than  $U_{\text{service}}$ , while lower values create a smoother demand curve. Note that  $U_i$  depends on  $\mu_1, \mu_2$  through  $SL_i$ .

**Simulation and equilibrium.** Similarly to Section 2, our goal is to find a sequential Nash equilibrium following the same sequence of decisions (prices/demand/pay/supply). However, simulating our queueing system for even one set of decisions is challenging. We solve it by computing the stationary distribution of the queueing system exactly as a linear system. To make this tractable, we cap the three queues in Fig. 3 to create a finite state space. To ensure that the cap does not influence our results while preserving tractability, we select the lowest cap that guarantees that the probability of one of the queues being full is less than 2% in all simulations.

Our highly optimized code can solve one simulation in two milliseconds. This speed is crucial because finding Nash equilibria is much harder than simulating one set of decisions<sup>13</sup>. They are not guaranteed to exist or be unique, and the decision space is too large for enumeration techniques. Instead, we find local Nash equilibria for each stage by simulating the best local responses until convergence. For example, in the first stage, at each iteration, the two firms simultaneously decide to increase or decrease their prices  $v_i$  by  $\epsilon$ . For each potential change, we recompute the subgame’s equilibria (demand, pay, supply), which are computed in a similar recursive way. The two firms then adjust prices in the direction that best increases their profit in the subgame equilibrium. Finally, an equilibrium is “found” when firms stop updating their decisions, or an equilibrium cycle is found when decisions begin to cycle.

If multiple equilibria or cycles exist, the one we converge to depends on the starting point. Therefore, we randomize the starting point and report the results. We also try to avoid trivial market collapse equilibria by choosing a space of starting points with high demand and supply. Making the approach tractable and stable was challenging, and we hope the simulator will be helpful to others.

## 5.2 Base case and order of events

Throughout this section, all figures share a consistent format: the x-axis of each plot varies a key parameter, while the curves represent the average equilibrium metric over 32 simulations with randomized initial conditions. Shaded ribbons indicate the interquartile range (25% to 75%) of the outcomes. The plots are organized in a grid, with the top row depicting the profit rates of the two firms,  $\pi_i$ , and the bottom row showing their worker pay,  $p_i$ . These two metrics are central to our analysis and the main focus of Theorem 1. The columns represent different simulation settings of interest.

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<sup>13</sup>The results presented in this paper required solving 178,016,152 queueing systems, corresponding to an average of 40,000 simulations to find each equilibrium.

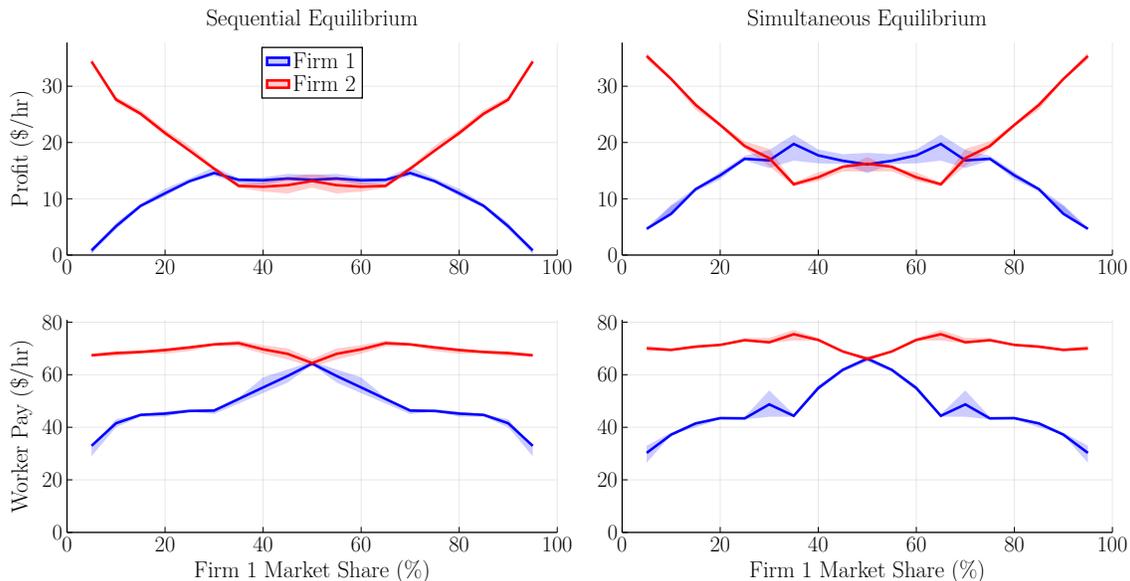


Figure 4: Sequential vs. Simultaneous Equilibria.

For example, Figure 4 shows our base case simulation results. The first column (“Sequential Equilibrium”) uses parameters that will serve as our base case for comparison. Here, we mimic the setting of Section 4.3 to verify that our simulator is consistent with our theory. In particular, this base case features a highly elastic supply, multi-homing workers, and non-competitive demand. (Details are given in Table 7 in Appendix F of the appendix.) Total demand ( $D_{\text{dedi-1}} + D_{\text{dedi-2}}$ ) is kept constant while we vary the market share of Firm 1 defined as  $D_{\text{dedi-1}} / (D_{\text{dedi-1}} + D_{\text{dedi-2}})$ .

The results align with the theoretical predictions of Fig. 2, despite the more intricate demand model used in the simulation. In particular, smaller firms pay workers less, in a way that resembles Theorem 1. For example, when the firms’ sizes are not too dissimilar, they have the same profit rate. The worker pay curve also exhibits a relatively flat segment for market shares between 10-30%, resembling the theoretical case where all small firms pay  $w_0$ . However, the curve is not perfectly flat because the finite supply elasticity in the worker MNL model means that pay is slightly increasing in size, even for small firms. The resulting diseconomy of scale in terms of profits is still pronounced: when Firm 2 is four times larger than Firm 1, its profit is only twice as large.

In the right column, we relax the four-stage assumption of our theoretical model. Here, we consider a two-stage sequential model where firms simultaneously choose price and pay first, while demand and supply equilibrate simultaneously in a second stage. Despite the additional complexity in supply-demand interactions, free-riding effects remain robust.

Intriguingly, we even observe *profit inversion*, where the smaller firm outperforms the larger one, a theoretical impossibility in the four-stage model. This occurs because, while smaller firms have less demand, their ability to pay workers less give them such a strong profit margin advantage that they become more profitable. In general, Fig. 4 illustrates the persistence of free-riding effects in this more realistic setting.

### 5.3 Varying the Supply Side

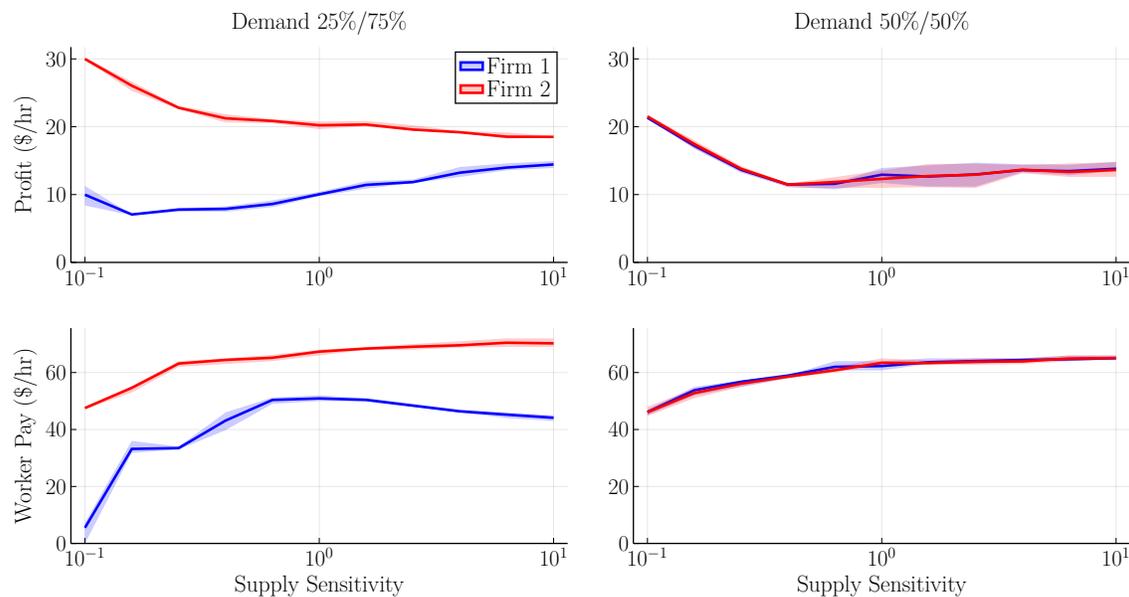


Figure 5: Impact of Supply Elasticity.

**Supply elasticity.** Next, Fig. 5 explores the effect of supply elasticity, which is parametrized by the supply sensitivity  $\alpha_{\text{supply}}$  (see Eq. (17)). In the left column, Firm 2 is three times larger than Firm 1, while they are the same size in the right column. We first notice that the smaller firm pays the workers less, regardless of the worker elasticity. The small firm’s profit is lower than the larger firm, but its profit margin is much higher. Interestingly, this effect is strongest when the elasticity is either particularly high or low. The high-elasticity case is similar to our theory; the smaller firm can free-ride completely. In the low elasticity case (which is typically unrealistic), some workers are willing to work for minuscule wages, which means that the smaller firm can pay particularly low while still having enough workers.

**Dedicated workers pools.** In practice, some workers are loyal and choose to work exclusively for one firm regardless of what the other firm pays. This can be due to the

impossibility of working for a given firm. For example, ridesharing drivers may only be registered with Lyft or Uber. It can also be due to loyalty incentives or an aversion to the complexity of setting up multi-apping. This behavior is not captured in our worker MNL model (Fig. 3), where the “dedicated” workers only choose to be dedicated because the other firm does not pay enough (i.e., they are rationally dedicated). Appendix F.2 of the appendix introduces such loyal workers to the simulation.

A natural hypothesis is that large dedicated pools reduce the free-riding effect by making firms less dependent on shared workers. However, this ignores the fact that dedicated workers typically wait longer because they have fewer jobs. The “thinner market” experienced by dedicated workers creates an earnings loss due to increased idleness, which means that shared workers are typically present in greater numbers in equilibrium. Indeed, the simulation shows that at high supply elasticity, dedicated workers participate in equilibrium only when the number of multi-apping workers is very limited. In this case, free-riding effects are still strong, and dedicated workers have a limited effect on the outcome. It is even possible to have *profit inversions* in equilibrium, where a firm that does not have access to dedicated workers can be better off than firms that do. In summary, dedicated workers are only helpful in markets where most workers are unwilling to multi-app.

#### 5.4 Varying the demand side

Section 4.1 shows that perfect competition results in market collapse, while Section 4.2 shows that market collapse can be avoided when one large firm has sufficient dedicated demand. Here, we explore the impact of dedicated demand in greater detail and examine another case of realistic competitive demand that can lead to profitable two-firm equilibria with strong free-riding effects.

To be able to represent both competitive and dedicated demand together, we keep the dedicated demand pools  $D_{\text{dedi-1}}$  and  $D_{\text{dedi-2}}$  of our previous model, who choose whether to participate as in Eq. (19). However, we also add a third competitive demand pool of size  $D_{\text{multiapp}}$ . This competitive demand also follows a multinomial logit (MNL) choice model with utility sensitivity  $\alpha_{\text{demand}}$ , but this time, we allow it to choose between the two firms based on expected utility. Therefore, the options of the MNL models are: choosing the outside option (and getting 0 utility), selecting Firm 1, or selecting Firm 2 (and getting the expected utility as in Eq. (18)). However, this typically leads to a destructive equilibrium because of price competition. We add a fourth option for this competitive demand pool to find more interesting equilibria. The  $D_{\text{multiapp}}$  customers can also choose to be *shared* and use both apps *at the same time*. In practice, this corresponds to customers who open both the Lyft and Uber apps and use the one with the closest driver available. In our model, the “closest driver” can be modeled as choosing a worker uniformly at random across all workers of the three queues and paying the price of the firm the worker workers for. The four options of the competitive demand pool are illustrated in Figure 6.

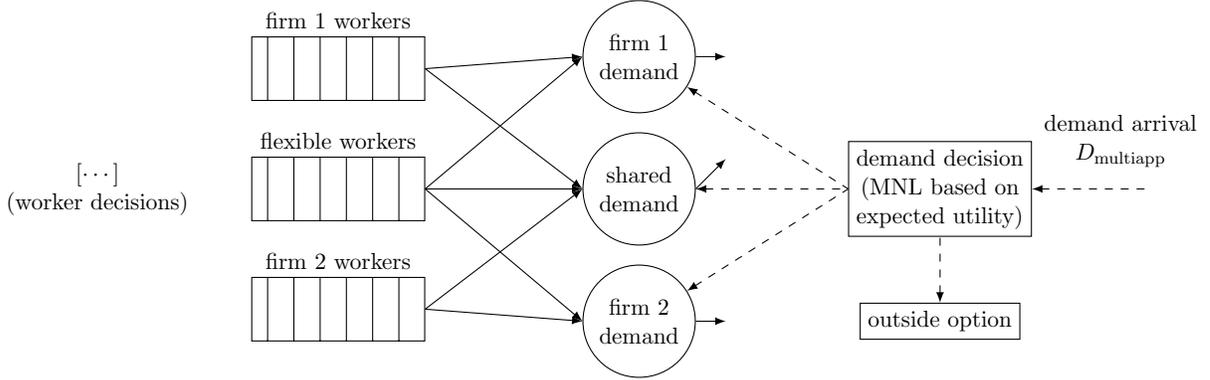


Figure 6: Modeling competitive demand

*Note:* The worker decision process mirrors Fig. 3. Competitive demand arrives as a Poisson process with rate  $D_{\text{multiapp}}$ . Customers have four types: they may (1) be dedicated to a firm, (2) use both firms, or (3) not request service (outside option). Dedicated customers are served by their chosen firm or leave if no workers are available. The probability that a shared customer choose a firm is proportional to its current number of available workers (e.g., as if the customer chose the ridesharing app with closest available driver). In equilibrium, customer choices follow a multinomial logit model based on expected utilities. The outside option’s utility is normalized to 0.

The expected utility of a shared customer is still evaluated using Eq. (18). However, the price is the average price experienced by shared customers (which matters if the two prices differ). Additionally, a shared customer can access all available drivers (even when some only work for one firm), which means that the service level  $SL_{\text{shared}}$  of this option is always the highest, which typically makes this option attractive. It significantly softens the race-to-the-bottom of price competition and creates the possibility of interesting equilibria where most competitive customers prefer the “shared” option, which prevents the price race to the bottom as firms cannot easily steal other firms’ customers by lowering prices.

Figure 7 illustrates the effects of this new demand pool. The total demand  $D_{\text{dedi-1}} + D_{\text{dedi-2}} + D_{\text{multiapp}}$  is constant and the x-axis is the fraction of demand in the dedicated pools,  $\frac{D_{\text{dedi-1}} + D_{\text{dedi-2}}}{D_{\text{dedi-1}} + D_{\text{dedi-2}} + D_{\text{multiapp}}}$ . For example, 0% dedicated demand corresponds to the fully competitive case. Each column shows a different distribution of the dedicated demand between the two firms (e.g., in the second column, we have  $\frac{D_{\text{dedi-1}}}{D_{\text{dedi-1}} + D_{\text{dedi-2}}} = 25\%$ ).

The symmetric case (Column 3) shows that the equilibria are more profitable when all the demand is competitive. This is because most competitive demand chooses to be shared in equilibrium. Shared demand is more efficient than dedicated demand because of pooling, with improved service levels and, therefore, attracting more customers at the same price. The destructive price competition equilibria are avoided.<sup>14</sup>

<sup>14</sup>Destructive equilibria may still exist, we just do not converge to them given our random starting point

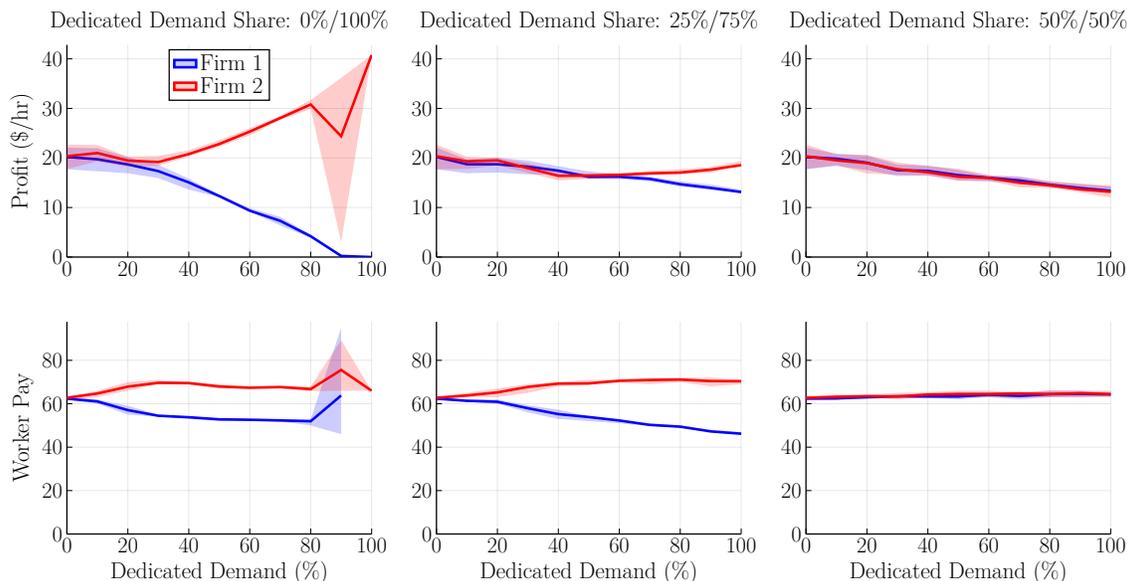


Figure 7: Impact of Demand Competition.

Columns 1 and 2 show that when one firm has less dedicated demand (Firm 1 is a “smaller” firm), it benefits from free-riding effects. This happens in a way similar to Theorem 1: pay is equal when firm sizes are comparable (low *Dedicated Demand* %, to the left of the x-axis). But with more dedicated demand, the smaller firm’s labor cost advantage increases as disparity grows, with an exception occurring when the dedicated demand is very high, and Firm 1 has none (in the first column, around 90% dedicated demand). In this case, the small firm, which only relies on competitive demand, does not have enough demand to be sustainable (for example, when *Dedicated Demand* = 100%, there is no more competitive demand, and Firm 1 has no demand).

We also studied the impact of the demand sensitivity to the service level, which can be crucial in markets such as ridesharing. The results, presented in Appendix F.3, are not particularly surprising: if customers demand higher service levels, firms must increase worker pay to attract more workers. In the worst case, this can lead to a destructive equilibrium. Interestingly, multi-apping competitive demand is particularly useful in this setting, as its service level advantage creates more robust equilibria. And free-riding effects are always present when a firm is smaller.

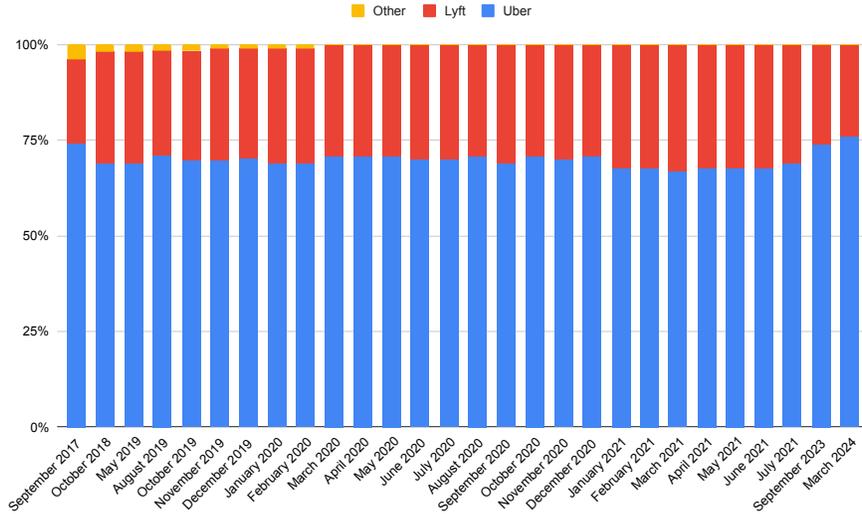


Figure 8: Trend of the market share of the leading ride-hailing companies in the US from Sept. 2017 to March. 2024. (Source: *Uber vs. Lyft: Who's tops in the battle of U.S. rideshare companies*, Second Measure, <https://secondmeasure.com/datapoints/rideshare-industry-overview/>)

## 6 Conclusions

A central feature of the gig economy is the existence of a shared labor pool. The primary result of this paper, Theorem 1, shows that maintaining this labor pool is similar to contributing to a common good, which leads to a free-riding phenomenon. In equilibrium, smaller firms are free-riding and pay workers a minimal amount, leaving the weight of maintaining the labor pool to the larger firms.

The second key result, Theorem 2, indicates that the shared labor pool may lead to a spillover effect for market formation. As long as one firm can jumpstart the gig economy, the gig economy allows smaller firms that could not be profitable on their own to thrive. This may relate to the observed growth of the gig economy in the U.S.: after the arrival of Uber and the jumpstart of the gig economy, there has been a flourishing diversity of services using independent contractors, including food delivery, grocery delivery, and package delivery (Sandusky 2021).

Free-riding is a robust outcome in any equilibrium with market formation, yet whether a market can form critically depends on the competition dynamics on the demand side. Proposition 3 demonstrates that perfect price competition leads to market collapse, 

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with sizable supply and demand.

nating the unique free-riding dynamics on labor. However, as long as one firm has enough loyal customers, Proposition 4 establishes that market collapse is never an equilibrium. These results provide insight into the observed competition dynamics in the ride-hailing industry, where studies (e.g., Chitla et al. 2023) show that a substantial proportion of customers prefer to single-home. For instance, Uber and Lyft remains a relatively stable duopoly (Fig. 8) over the years, despite Uber holding a significantly larger market share.

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# Appendices

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## A Worker strategies

Workers have two decisions:

1. Whether to join in the gig economy, which is captured by the worker arrival rate  $\lambda$ . This is discussed in Appendix A.2.
2. Conditional on joining, whether to accept a certain job, which is captured by a job acceptance rule that depends on the worker pay. This is discussed in Appendix A.1.

### A.1 Micro-structure model

We model the decision process of an individual worker as a Markov decision process as follows: Workers are in one of three states, denoted  $s$ , as shown in Table 1:

$s$	Description
-1	out of the worker pool
0	waiting for work in the worker pool
1	working

Table 1: Worker states

Workers earn their reservation wage  $w_0$  for any time spent outside the worker pool. Jobs have the same average duration of  $T = 1/\lambda_0$ , where the job duration is exponentially distributed and  $\lambda_0$  is the rate at which jobs finish. After accepting a job, a worker has to complete it. We assume workers rejoin the pool upon completion of a job. (They can immediately exit the pool once they rejoin it.) Firm  $j$  offers jobs to the worker at an independent Poisson rate  $\lambda_j$ ,  $j = 1, \dots, N$ , and job completion events are Poisson at rate  $\lambda_0$ . Without loss of generality, we assume the overall rate of events  $\lambda_0 + \sum_{j=1}^N \lambda_j = 1$ , which implies the mean interarrival time between events is one unit of time. Let the random variable  $\omega$  denote the type of arrival event (i.e.,  $\omega = j$  if the arrival event is a job offer from firm  $j$ ) and note that  $P(\omega = j) = \lambda_j$ ,  $j = 0, \dots, N$ .

We assume the worker makes decisions only when events occur, so their decision-making process is embedded at these arrival time epochs, which we index by  $t$ . Formally, let  $a$  denote the action (decision) of the worker,  $A(s, \omega)$  denote the set of feasible actions when a worker is in state  $s$  and faced with event  $\omega$ . The possible actions are defined in Table 2:

The feasible action sets are

$$A(-1, \omega) = \{-1, 0\} \tag{20}$$

$$A(0, \omega) = \{-1, 0, \omega\} \tag{21}$$

$$A(1, 0) = \{0\} \tag{22}$$

$a$	Description
-1	leave the worker pool
0	join (or stay in) the worker pool
$j$	accept a job from firm $j$ , $j = 1, \dots, N$

Table 2: Worker actions

Each state-action pair  $(s, a)$  generates an expected reward  $r(s, a)$  defined by

$$r(s, a) = \begin{cases} w_0 & s = -1, 0; a = -1 \\ p_j/\lambda_0 & s = 0; a = j \\ 0 & s = -1, 0; a = 0 \\ 0 & s = 1; a = 0 \end{cases}$$

This implies workers are paid for an accepted job at the time they decide to accept, where  $p_j$  is the pay rate of firm  $j$ ,  $j = 1, \dots, N$  and  $1/\lambda_0$  is the average duration of a job. When  $j = 0$ ,  $p_0 = 0$ , which can be thought of as a hypothetical job paying zero to the worker. We sometimes also use the notation  $p_\omega$  to represent how much the worker gets paid when accepting a job from event  $\omega$ . That is,  $p_\omega = p_j$  when  $\omega = j$ . Similarly, the reservation wage rate  $w_0$  is earned when a worker decides to leave the pool and remains outside the pool until the next decision epoch (event arrival), an expected duration of one unit of time. For an average cost expected value problem, accounting for these expected rewards at the decision epoch is without loss of generality.

The non-zero state transition probabilities,  $q_{ss'}(a) = P(s_{t+1} = s' | s_t = s, a)$ , are

$$\begin{aligned} q_{-1,-1}(-1) &= 1, & q_{-1,0}(0) &= 1 \\ q_{0,-1}(-1) &= 1, & q_{0,0}(0) &= 1, & q_{0,1}(j) &= 1, \\ q_{1,0}(0) &= \lambda_0, & q_{1,1}(0) &= 1 - \lambda_0 \end{aligned} \quad (23)$$

That is, a decision to join or quit the pool changes the state deterministically accordingly, and a decision to accept a job deterministically puts the worker in the working state. The only probabilistic transitions are when a worker is working, in which case they finish work when a job completion event occurs and continue working otherwise.

The worker's objective is to maximize their expected average long-run reward. This maximization is achieved by solving the following average-cost dynamic programming optimality condition

$$\gamma + h(s) = E_\omega \left[ \max_{a \in A(s, \omega)} \left\{ r(s, a) + \sum_{s'} q_{ss'}(a) h(s') \right\} \right], \quad s = -1, 0, 1. \quad (24)$$

where  $\gamma$  is the long-run reward rate and  $h(s)$  is the differential reward of state  $s$ . For the feasible action sets, rewards and state transition probabilities defined above, this reduces to solving:

$$\gamma + h(-1) = \max \{w_0 + h(-1), h(0)\} \quad (25)$$

$$\gamma + h(0) = E_\omega [\max \{w_0 + h(-1), h(0), p_\omega/\lambda_0 + h(1)\}] \quad (26)$$

$$\gamma + h(1) = \lambda_0 h(0) + (1 - \lambda_0)h(1) \quad (27)$$

If the market is in equilibrium, it must be true that workers who are out of the pool are indifferent between staying out or joining. Likewise, workers who are waiting must be indifferent between exiting the pool and continuing waiting. This means both alternative actions in (25) of staying out or joining must have equal value, and similarly the two actions of continuing to wait or exiting in (26) must have equal value. These conditions reduce to satisfying the following equilibrium condition in addition to the optimality condition (24).

$$w_0 + h(-1) = h(0) \quad (28)$$

Lastly, we also require that the solution satisfy the equilibrium condition (1), which recall states that the opportunity cost of waiting for work must equal the excess earnings on that work.

The next proposition provides the optimal solution:

**Proposition 6.** *The unique optimal solution to both the equilibrium conditions (28) and (1), and the optimality condition (24) is  $\gamma^* = w_0$ ,  $h^*(-1) = 0$ ,  $h^*(0) = w_0$  and  $h^*(1) = -w_0(1 - \lambda_0)/\lambda_0$ . Moreover, the worker's optimal decision while in the worker pool is to accept a job offer from firm  $j$  iff  $p_j \geq w_0$ .*

*Proof.* Proof. It is easy to verify by direct substitution that this solution satisfies the equilibrium condition (28) and optimality conditions (25) and (27). This leaves only the optimality condition (26). To analyze it, note that for  $h = h^*$ ,  $w_0 + h(-1) = h(0) = w_0$ , so the inner maximization in (26) reduces to

$$\max \{w_0, p_\omega/\lambda_0 + h^*(1)\}$$

Substituting  $h^*(1) = -w_0(1 - \lambda_0)/\lambda_0$  for the second term in the max (accepting the job  $\omega$ ), we see that

$$p_\omega/\lambda_0 + h^*(1) = \frac{1}{\lambda_0}(p_\omega - w_0(1 - \lambda_0)),$$

where the right-hand side above is greater than or equal to  $w_0$  iff  $p_\omega \geq w_0$ . This proves the claim about the worker's optimal decision while in the worker pool.

Now, let  $\lambda^* = \sum_{j:p_j \geq w_0} \lambda_j$  denote the rate of jobs from firms offering at least  $w_0$  and define the average accepted price by

$$P^* = \left( \sum_{j:p_j \geq w_0} \lambda_j p_j \right) / \lambda^*$$

Then (26) becomes

$$\gamma^* + h^*(0) = \lambda^*(P^*/\lambda_0 + h^*(1) + (1 - \lambda^*)w_0)$$

Substituting the values of  $\gamma^*$  and  $h^*$  and simplifying, we have

$$w_0 \frac{1}{\lambda^*} = (P^* - w_0) \frac{1}{\lambda_0}.$$

But note that  $1/\lambda^*$  is simply the worker's waiting time for a job they are willing to accept (under their optimal policy), so the left-hand side is the worker's cost of waiting for work. And the right-hand side is the total excess pay (pay above the reservation wage) a worker receives from an accepted job. Hence, this condition simply says that the worker's waiting cost equals the excess pay generated from waiting, which is precisely the equilibrium condition (1).  $\square$

## A.2 Macro market model

The number of workers that accept jobs from firm  $i$  is given by

$$\lambda_i = \frac{\mu_i}{\mu_{\geq w_0}} \lambda(\mathbf{p}) \mathbb{1}_{p_i \geq w_0} \quad (29)$$

where  $\lambda(\mathbf{p})$  is given by Eq. (4).

## B Key results

In this section, we first derive results for the stage of wage-setting — the subgame equilibrium of worker pay, while treating the demand rates  $\boldsymbol{\mu}$  and prices  $\mathbf{v}$  and exogenously given. Importantly, Appendix B.1 introduces a reformulation of the firm's profit maximization problem, which builds intuition and serves as the proof technique for the rest of proof. Then we prove that in equilibrium, all firms with  $v_i^* > w_0$  will participate. Finally, we introduce the proof of Theorem 1, which is also derived based on the reformulation.

### B.1 Problem reformulation and proof techniques

The first step of the proof of Theorem 1 is to introduce an intuitive reformulation of the profit maximization problem (5). While firm  $i$ 's decision is to set the worker pay  $p_i$ , we use a change of variables and let firm  $i$  choose the quantity  $r_i$  instead, which is defined as:

$$r_i \triangleq (v_i - p_i) \mu_i t_i \quad (30)$$

The first term  $v_i - p_i$  is firm  $i$ 's profit rate per job hour, while the second term  $\mu_i t_i$  is the maximum number of job hours that the firm can fulfill per hour in expectation. Therefore  $r_i$

is the maximum profit rate that firm  $i$  could make given the worker pay  $p_i$ . The maximum profit rate must be no greater than the firm's surplus  $s_i = (v_i - w_0)\mu_i t_i$ ; when firm  $i$  chooses to pay the lowest possible rate  $w_0$ ,  $r_i = s_i$ . In other words, choosing  $p_i \geq w_0$  is equivalent to choosing  $r_i \leq s_i$ . With this change of variables, the equilibrium worker arrival rate  $\lambda(\mathbf{p})$  given by Eq. (4) then can be expressed as a function of  $\mathbf{r} = (r_1, \dots, r_N)$ :

$$\lambda(\mathbf{r}) = \left( \sum_{i, r_i \leq s_i} \mu_i \right) \left( 1 - \frac{w_0}{\sum_{i, r_i \leq s_i} (s_i - r_i)} \right)^+ \quad (31)$$

The first term,  $\sum_{i, r_i \leq s_i} \mu_i$ , is simply the total demand rate of the acceptable jobs. Therefore, the second term is the *job completion rate of the gig economy*. It is the probability that a job is fulfilled by a worker, which is the same for all firms. We denote the job completion rate as  $\rho$  and formally introduce its definition:

$$\rho(\mathbf{r}) \triangleq \left( 1 - \frac{w_0}{\sum_{i, r_i \leq s_i} (s_i - r_i)} \right)^+ = \left( 1 - \frac{w_0}{\sum_i (s_i - r_i)^+} \right)^+ \quad (32)$$

With this notation, firm  $i$ 's hourly profit rate  $\pi_i$  from (5) takes a simpler form:

$$\pi_i = r_i \rho(\mathbf{r}) \mathbb{1}_{r_i \leq s_i} \quad (33)$$

**Common good.** The reformulated profit function (33) illustrates the fundamental trade-off faced by a firm of size  $s_i$ . Choosing the worker pay is equivalent to splitting the surplus (size)  $s_i$  into two quantities,  $r_i$  and  $(s_i - r_i)$ . A higher maximum profit rate  $r_i$  is clearly desirable. But as a firm increases  $r_i$ , the term  $(s_i - r_i)$  in the denominator of  $\rho(\mathbf{r})$  declines and the firm contributes less to maintaining the worker pool. This in turn reduces the completion rate of the entire economy and reduces the firm's realized profit rate. The firm must therefore decide how to allocate its surplus  $s_i$  between increasing its maximum profit rate  $r_i$  and contributing to maintaining the worker pool,  $(s_i - r_i)$ , which is a common good.

**Free-riding.** This trade-off is different for small and large firms. A firm with a small size  $s_i$  has a limited impact on  $\rho$  and therefore prefers to allocate more of its surplus to its profit rate  $r_i$ . Conversely, a larger firm has a greater impact on the completion rate  $\rho$  and therefore has more of an incentive to give up some of its maximum profit rate  $r_i$  to increase the completion rate. The striking feature of Theorem 1 is the degree of this effect; the increased worker pay of large firms is so high that they end up making the same total profit regardless of their size. On the other hand, small firms choose to entirely *free-ride* on larger firms, and not invest any surplus in the gig economy (i.e.  $r_i = s_i$ ).

**Maintaining the gig economy.** Theorem 1 states that  $\sum_i s_i > w_0$  is required for the gig economy to exist. With the reformulation, this condition becomes even more intuitive: for the job completion rate  $\rho(\mathbf{r})$  in Eq. (32) to be positive, it requires the total contribution

of the firms' surplus to the common good  $\rho$  to be higher than  $w_0$  (i.e.  $\sum_i (s_i - r_i)^+ > w_0$ ), so that workers are paid enough to choose to stay in the gig economy. Therefore, the total size of the firms must be higher than  $w_0$  to meet this requirement. An interesting observation is that, if  $\sum_i s_i$  is greater than but close to  $w_0$ , most of the gig economy's surplus will be allocated to maintain the common worker pool, which means firms have little opportunity to generate their own profits.

**Proof intuition.** The reformulation also sheds light on why larger firms that pay strictly above  $w_0$  all have the same hourly profit rate regardless of their sizes (Result 2 in Theorem 1). For all firms that do not free-ride (i.e.  $r_i < s_i$ ), the gradient of the job completion rate  $\rho$  in Eq. (32) with respect to  $r_i$  is identical, as the denominator in Eq. (32) is linear in  $r_i$  if  $r_i < s_i$ . Therefore, the gradient of the profit  $\pi_i(r_i; r_{-i})$  with respect to  $r_i$  is also symmetric for all firms that do not free-ride. Since the best response is solved by setting the gradient to zero for non-free-riding firms, we should expect that all these firms share the same  $r_i$  and the same hourly profit rate  $\pi_i$ . A formal proof and more details are provided in Proposition 7 in the appendix.

In this section, we characterize the Nash equilibrium  $(p_1, \dots, p_N)$  among  $N$  firms, holding the demand side as fixed. As mentioned in Appendix B.1, the analysis is based on a reformulation of the problem from the decision space of the worker pay  $(p_1, \dots, p_N)$  to the potential profit rate  $(r_1, \dots, r_N)$ . Please refer to Eq. (30) and Eq. (32) for details and Appendix B.1 for more intuition behind the reformulation.

## B.2 Firms' participation (Proof of Proposition 2)

*Proof.* Proof. We prove by showing that the following three statements cannot be true at the same time for a strategy profile  $\mathbf{r} = (r_1, r_2, \dots, r_N)$ :

- It is a Nash equilibrium.
- The resulting worker participation rate from  $\mathbf{r} = (r_1, r_2, \dots, r_N)$  has market formation, i.e.  $\lambda > 0$ .
- There exists a firm  $j$  with  $v_j > w_0$ , but firm  $j$  does not participate in the gig economy in this strategy profile, i.e.  $r_j > s_j > 0$ .

We prove it by contradiction.

Suppose not. Then if a market can form ( $\lambda > 0$ ), it must be true that there are some firms participating ( $\mu_{\geq w_0} > 0$ ) and the job completion rate  $\rho > 0$ . Suppose in the meantime, firm  $j$  does not participate ( $r_j > s_j$ ). Then by the equilibrium condition Eq. (32),

$$\rho(r_j > s_j; r_{-j}) = \left(1 - \frac{w_0}{\sum_{i \neq j} (s_i - r_i)^+}\right)^+ > 0 \quad (34)$$

Now suppose firm  $j$  deviates and participates by letting  $r_j = s_j$ . Then the job completion rate does not change:

$$\rho(r_j = s_j; r_{-j}) = \left(1 - \frac{w_0}{(s_j - s_j)^+ + \sum_{j \neq i} (s_i - r_i)^+}\right)^+ = \rho(r_j > s_j; r_{-j}) > 0 \quad (35)$$

and firm  $j$ 's profit will be strictly improved :

$$\pi_i = s_j \rho(r_j = s_j; r_{-j}) > 0 \quad (36)$$

In other words, it is not firm  $j$ 's best response to stay out of the gig economy. Hence,  $(r_1, r_2, \dots, r_N)$  cannot be a Nash equilibrium. Contradiction.  $\square$

### B.3 Free-riding and equilibrium existence (Proof of Theorem 1 and Theorem 2)

In the following proposition, we comprehensively characterize the Nash equilibrium of worker pay, detailing all possible types of equilibria, the conditions under which each type exists, their uniqueness, and the mathematical formulas that underpin the properties described in Theorem 1.

**Proposition 7.** *Given  $N$  firms and their sizes  $s_1 \geq s_2 \geq \dots \geq s_N > 0$ , there only exist two types of equilibria:*

1. *No market formation ( $\lambda^* = 0$ ), and no firm makes a profit. This type of equilibrium exists if and only if  $\max_i \{s_i\} \leq w_0$ .*
2. *Market formation ( $\lambda^* > 0$ ), and all firms make positive profits. Moreover, the market formation equilibrium is uniquely defined by*

$$1 - \rho^* = \frac{w_0}{\sum_i (s_i - \frac{w_0 \rho^*}{(1 - \rho^*)^2})^+} \quad (37)$$

and

$$r_i^* = \min\left\{\frac{w_0 \rho^*}{(1 - \rho^*)^2}, s_i\right\}, \quad \forall i \quad (38)$$

*This is the equilibrium described in Theorem 1; it exists if and only if  $\sum_i s_i > w_0$ .*

*Proof.* Proof. We prove the two cases in Proposition 7 sequentially:

- **Case 1:** No market formation ( $\lambda^* = 0$ ).
- **Case 2:** Market formation ( $\lambda^* > 0$ ).

For each case, we demonstrate that the described equilibria are Nash equilibria (NE) and establish their existence and uniqueness under the specified conditions. By Proposition 2, these two cases fully characterize all possible equilibria.

## No Market Formation (Item 1)

We prove that  $\max_i \{s_i\} \leq w_0$  is both a necessary and sufficient condition for an equilibrium with  $\lambda = 0$ .

### Sufficient Condition

We show that when  $\max_i \{s_i\} \leq w_0$ , an NE with  $\lambda = 0$  must exist.

Suppose  $\max_i \{s_i\} \leq w_0$ . If all other firms do not participate ( $r_j > s_j$  for all  $j \neq i$ ), the job completion rate  $\rho(\mathbf{r})$  from Eq. (32) depends only on firm  $i$ 's decision  $r_i$ :

$$\rho(\mathbf{r}) = \rho(r_i) = \left(1 - \frac{w_0}{(s_i - r_i)^+}\right)^+. \quad (39)$$

Clearly,  $\rho(r_i)$  is decreasing in  $r_i$ . Since no firm's size exceeds  $w_0$ , including firm  $i$ , it follows that:

$$\rho(r_i) \leq \rho(0) = \left(1 - \frac{w_0}{s_i}\right)^+ = 0. \quad (40)$$

Thus, when all other firms do not participate, firm  $i$  cannot achieve a positive profit by participating. This reasoning applies to all firms, so no firm will participate, resulting in no market formation ( $\lambda = 0$ ).

### Necessary Condition

We now show that if  $\max_i \{s_i\} > w_0$ , an equilibrium with  $\lambda = 0$  cannot exist.

Let  $\max_i \{s_i\} = s_1 > w_0$ . By the equilibrium condition Eq. (32), the job completion rate  $\rho(\mathbf{r})$  is weakly decreasing in  $r_i$  for all  $i$  since less surplus is left in the common pool to maintain the common good. Therefore:

$$\rho(\mathbf{r}) \geq \rho(r_1; r_2 = s_2, \dots, r_N = s_N) = \left(1 - \frac{w_0}{(s_1 - r_1)^+}\right)^+. \quad (41)$$

Since  $s_1 > w_0$ , there exists  $0 < r_1 \leq s_1$  such that  $\rho(\mathbf{r}) > 0$ . Hence, firm 1 can form a market on its own and is strictly better off by participating. This contradicts the assumption of no market formation ( $\lambda = 0$ ). Thus,  $\max_i \{s_i\} \leq w_0$  must hold for an equilibrium with  $\lambda = 0$  to exist.

## Market Formation (Item 2)

In this proof, we characterize the Nash equilibrium (NE) under the conditions described in Item 2, Proposition 7. We prove the following:

- The sufficient and necessary conditions for the existence of an NE with  $\lambda > 0$ .
- The uniqueness of this equilibrium and its properties as described in Theorem 1.

### Sufficient Condition

We first show that  $\sum_i s_i > w_0$  is a sufficient condition for the existence of an NE with  $\lambda > 0$ . This NE satisfies Eq. (37) and Eq. (38).

Consider the best response of firm  $i$  given other firms' strategies,  $r_j$  for  $j \neq i$ . For  $r_i \leq s_i$ , the first-order derivative of the profit function in Eq. (33) is:

$$r_i \cdot \frac{\partial \rho}{\partial r_i} + \rho. \quad (42)$$

The derivative of  $\rho$  with respect to  $r_i$  is:

$$\frac{\partial \rho}{\partial r_i} = w_0(s_i - r_i + \sum_{j \neq i} (s_j - r_j)^+)^{-2}(-1), \quad (43)$$

$$= w_0 \left( \frac{1 - \rho}{w_0} \right)^2 (-1), \quad (44)$$

$$= -\frac{(1 - \rho)^2}{w_0} < 0. \quad (45)$$

The second-order derivative is:

$$\frac{\partial^2 \rho}{\partial r_i^2} = -2w_0(s_i - r_i + \sum_{j \neq i} (s_j - r_j)^+)^{-3} < 0. \quad (46)$$

Since the profit function is strictly concave, the best response is obtained by setting Eq. (42) to zero:

$$r_i = \frac{w_0 \rho}{(1 - \rho)^2}. \quad (47)$$

If this solution satisfies  $r_i \leq s_i$ , it is the firm's best response. Otherwise, the profit is strictly increasing in  $r_i$ , and the best response is  $r_i = s_i$ . Thus, the best response of firm  $i$  is:

$$r_i^*(r_{-i}) = \min \left\{ \frac{w_0 \rho}{(1 - \rho)^2}, s_i \right\}. \quad (48)$$

At equilibrium, when all firms choose their best responses, the equilibrium job completion rate  $\rho^*$  must satisfy:

$$1 - \rho^* = \frac{w_0}{\sum_i (s_i - \frac{w_0 \rho^*}{(1 - \rho^*)^2})^+}. \quad (49)$$

Using Eq. (49), it is straightforward to verify that a unique solution for  $\rho^* > 0$  exists if and only if  $\sum_i s_i > w_0$ .

### Necessary Condition

We now show that  $\sum_i s_i \leq w_0$  is a necessary condition for the absence of an NE with  $\lambda > 0$ . By the equilibrium condition in Eq. (32):

$$\rho(\mathbf{r}) = \max \left\{ 0, 1 - \frac{w_0}{\sum_i (s_i - r_i)^+} \right\}. \quad (50)$$

If  $\sum_i s_i \leq w_0$ , then for all strategies  $r_i$ , it follows that:

$$\rho(\mathbf{r}) \leq \max \left\{ 0, 1 - \frac{w_0}{\sum_i s_i} \right\} = 0. \quad (51)$$

Thus, when  $\sum_i s_i \leq w_0$ , the total firm size is insufficient to maintain a positive worker pool, and no NE with  $\lambda > 0$  exists.

### Verification of Properties

We now verify that the NE with  $\lambda > 0$  satisfies the properties in Theorem 1:

1. *It is the only NE with  $\lambda > 0$ .* This is about the uniqueness of the solution defined by Eq. (37) and Eq. (38), which has been proved in the paragraph below Eq. (49) and illustrated by Fig. 9. The idea is that the right-hand side Eq. (37),  $w_0 / \sum_i (s_i - w_0 \rho / (1 - \rho)^2)^+$ , is strictly increasing in  $\rho$ . As  $\frac{w_0 \rho}{(1 - \rho)^2}$  (which is strictly increasing in  $\rho$ ) approaches  $\max_i \{s_i\}$ , the denominator goes to zero, and the right-hand side goes to infinity. Therefore, the right-hand side and the left-hand side of Eq. (37) have one and only one intersection with  $\rho > 0$  when  $w_0 / \sum_i s_i > 1$ .
2. *There exists a threshold size  $s_\sigma$ , above which firms pay strictly higher than  $w_0$  and all of these firms have the same profit rate  $\pi_i$ .* This can be shown by observing the form of the equilibrium profit rate Eq. (38): there is a threshold value  $s_\sigma = w_0 \rho^* / (1 - \rho^*)^2$ , which is a function of the sizes of all firms. When a firm's size is above this threshold ( $s_i > s_\sigma$ ),  $r_i^* = s_\sigma > s_i$ , implying that  $p_i^* > w_0$  and all firms have the same profit.
3. *All firms at or below  $s_\sigma$  pay  $w_0$ .* Related to the last item, when a firm's size is below the threshold ( $s_i \leq s_\sigma$ ),  $r_i^* = s_i \leq s_\sigma$ , implying that  $p_i^* = w_0$ .  $\square$

By combining the sufficient and necessary conditions, we have established the existence, uniqueness, and properties of the Nash equilibrium with  $\lambda > 0$  as stated in Proposition 7.

$\square$

$\square$

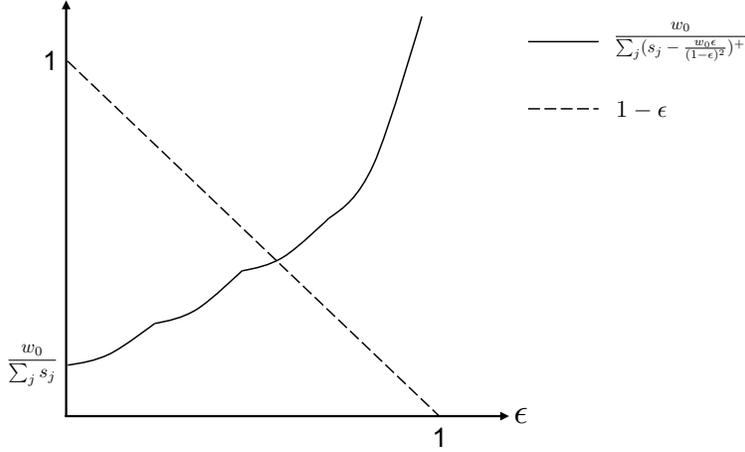


Figure 9: Illustration of the equilibrium job completion rate  $\rho$

## C Competition models and equilibrium existence

### C.1 Perfectly competitive demand (Proof of Proposition 3)

Consider the case where the competing firms sell the same undifferentiated service in the same output market. Without loss of generality, we normalize the job duration  $t_i = 1$  for all firms. Consumers care only about price and always chose the firm with the lowest price. We will also make a technical assumption that there is a minimal job completion rate,  $\underline{\rho}$ , that is acceptable to the consumers in the sense that if the job completion rate drops below  $\underline{\rho}$ , consumers' utility drops to zero and they chose other non-gig-economy alternatives. We then prove Proposition 3 by letting  $\underline{\rho} = 0$

More precisely, under perfect competition, there can only be one price in the market, which is the lowest price among all firms, which we denote as  $v$ . The total demand in the market,  $\mu$ , can therefore be characterized as a function of the market-level price  $v$  and the job completion rate  $\rho$ :

$$\mu(v, \rho) = \begin{cases} \mu(v), & \rho \geq \underline{\rho} \\ 0, & \rho < \underline{\rho} \end{cases} \quad (52)$$

We assume that  $\mu(v)$  is a continuous and decreasing function of the price  $v$ . When multiple firms offer the lowest price, the demand is split equally among them. In other words, if there are two firms with price  $v$ , then each firm has a demand of  $\mu(v, \rho)/2$ .

Under perfect competition, competition leads to a price war; to win the whole market, firms always have a tendency to cut prices. But to cut prices, worker pay has to be reduced as well to keep a non-negative profit margin. Thus, firms keep depleting the worker pool

until the point that the job completion rate (service quality) is so low that it reaches the minimum acceptable rate, i.e.  $\rho_i = \underline{\rho}$  for all firms. (This is formally proven in Lemma 2 in the appendix.) Thus, when  $\underline{\rho} = 0$ , the only equilibrium is market collapse.

**Notations and definitions.** Before diving into the proof, we introduce the following definition to make the analysis more intuitive. In this proof, we will frequently use Eq. (32) between the job completion rate  $\rho$  and the price and the worker pay. For convenience we repeat here: When  $N$  firms with demand  $(\mu_1, \dots, \mu_N)$ , prices  $(v_1, \dots, v_N)$  and worker pay  $p_1, \dots, p_N$  share a pool of workers, the job completion rate of the shared pool is given by:

$$\rho = \left(1 - \frac{w_0}{\sum_i (s_i - r_i)^+}\right)^+ = \left(1 - \frac{w_0}{\sum_i \mu_i (p_i - w_0)^+}\right)^+ \quad (53)$$

This rightmost equality is by the definition that of  $s_i = (v_i - w_0)\mu_i t_i$  and  $r_i = (v_i - p_i)\mu_i t_i$  in Eq. (11) and Eq. (30). When each firm has a dedicated worker pool, firm  $i$ 's job completion rate is given by

$$\rho_i = \left(1 - \frac{w_0}{(s_i - r_i)^+}\right)^+ = \left(1 - \frac{w_0}{\mu_i (p_i - w_0)^+}\right)^+ \quad (54)$$

The term  $t_i$  does not appear as we have normalized  $t_i = 1$  in Appendix C.1. Note that although we generally do not use  $r_i$  and  $s_i$  in this section of the proof (since the demand is now a function of both  $v$  and  $\rho$ ), Eq. (53) and Eq. (54) remains to hold since they merely characterize the job completion rate given the demand  $\mu_i$  and worker pay  $p_i$ . As we go into the analysis of the firms' strategies, it will become clear how  $\mu_i$  can be dependent on  $\rho$  and how we deal with it.

Moreover, regardless of whether the worker pool is shared or not, there cannot exist a Nash equilibrium with a firm making a positive profit margin (i.e.  $v_i > p_i$ ) and a positive demand at the same time. Otherwise, an opponent can always increase its profit by undercutting the firm's price and stealing the demand of the whole market. Below we formally state the results and proof:

**Lemma 1.** *Under perfect competition, there does not exist a Nash equilibrium with a firm making a positive profit. In other words, both firms have a profit  $\pi_i = 0$ ,  $i = 1, 2$ .*

*Proof.* Proof. We prove this by contradiction. Suppose there is a firm (firm  $i$ ) making a positive profit. Then this requires three things to be true:

1. Firm  $i$ 's worker pool has a job completion rate  $\rho_i \geq \underline{\rho}$ .
2. Firm  $i$ 's price is (among) the lowest in the market. Otherwise, firm  $i$  does not get any demand.
3. Firm  $i$ 's price is strictly higher than its worker pay, i.e.  $v_i > p_i$ . Otherwise, firm  $i$ 's profit margin will be negative.

Since there can only be one price in a perfectly competitive market, firm  $i$ 's opponent (firm  $-i$ ) must either have the same price as firm  $i$  or stay out of the gig economy. We show that in either case, firm  $-i$  can strictly benefit from undercutting firm  $i$ 's price and making firm  $i$ 's demand drop to zero.

First, in both settings, if firm  $-i$  does not join the gig economy in the first place, then it can strictly increase its profit by joining and let  $v_{-i} = v_i - \epsilon$  and  $p_{-i} = p_i$ . Since firm  $i$ 's price is strictly higher than its pay, firm  $-i$  can get a positive profit and the demand for the whole market by undercutting firm  $i$ 's price. In this case, firm  $i$  loses all its demand and cannot make a positive profit.

Next, we consider the scenario in which both firms participate.

If both firms participate in the gig economy sharing the labor pool, and firm  $i$  makes a positive profit, then it must be true that

$$\rho_i = \rho_{-i} = 1 - \frac{w_0}{\mu(v)/2 \cdot (p_i - w_0) + \mu(v)/2 \cdot (p_{-i} - w_0)} \geq \underline{\rho}$$

This is by Eq. (53). Let  $\bar{p} = (p_i + p_{-i})/2$ . Then this is equivalent to

$$1 - \frac{w_0}{\mu(v)(\bar{p} - w_0)} \geq \underline{\rho} \Leftrightarrow \bar{p} \geq \frac{w_0}{\mu(v)(1 - \underline{\rho})} + w_0 \quad (55)$$

The two firms must have the same price, which we denote as  $v_1 = v_2 = v$ . Then the total profit of the whole market is given by

$$\pi = \pi_i + \pi_{-i} \quad (56)$$

$$= \frac{\mu(v)}{2}(v - p_i)\rho_i + \frac{\mu(v)}{2}(v - p_{-i})\rho_{-i} \quad (57)$$

$$= \left( \frac{\mu(v)}{2}(v - p_i) + \frac{\mu(v)}{2}(v - p_{-i}) \right) \left( 1 - \frac{w_0}{\mu(v)/2 \cdot (p_i - w_0) + \mu(v)/2 \cdot (p_{-i} - w_0)} \right) \quad (58)$$

$$= \mu(v) \left( v - \frac{p_i + p_{-i}}{2} \right) \left( 1 - \frac{w_0}{\mu(v)((p_i + p_{-i})/2 - w_0)} \right) \quad (59)$$

$$= \mu(v)(v - \bar{p}) \left( 1 - \frac{w_0}{\mu(v)(\bar{p} - w_0)} \right) \quad (60)$$

where  $\bar{p} = (p_i + p_{-i})/2$ . In other words, the total profit of the whole market is determined by the market level price  $v$  and the average worker pay between the firms. Since firm  $i$  makes a strictly positive profit, it must be true that  $\pi_{-i} = \pi - \pi_i < \pi$ .

Now we show that by undercutting the price of firm  $i$ , firm  $-i$  can achieve a profit that is strictly higher than  $\pi_{-i}$ . Let firm  $-i$  deviate to the following strategy:

$$v_{-i} = v - \epsilon \quad (61)$$

Moreover, let firm  $-i$  choose the worker pay by solving the monopoly's profit maximization problem:

$$\max_p \mu(v - \epsilon) \cdot (v - \epsilon - p) \left(1 - \frac{w_0}{\mu(v - \epsilon)(p - w_0)}\right) \quad (62)$$

s.t.

$$p \geq \frac{w_0}{\mu(v - \epsilon)(1 - \underline{\rho})} \quad (63)$$

Eq. (63) is to make sure firm  $-i$ 's job completion rate,  $(1 - \frac{w_0}{\mu(v - \epsilon)(p - w_0)})$ , to be at least  $\underline{\rho}$ . In other words, let firm  $-i$ 's worker-side strategy be:

$$p'_{-i} = \arg \max_{p \geq \frac{w_0}{\mu(v - \epsilon)(1 - \underline{\rho})}} \mu(v - \epsilon) \cdot (v - \epsilon - p) \left(1 - \frac{w_0}{\mu(v - \epsilon)(p - w_0)}\right) \quad (64)$$

and denote the corresponding profit as  $\pi'_{-i}$ . Since  $\pi'_{-i}$  is the profit at the optimal worker pay in this case, it must be no less than the corresponding profit for any worker pay that satisfies  $p \geq \frac{w_0}{\mu(v - \epsilon)(1 - \underline{\rho})}$ ; by Eq. (55),  $\bar{p}$  qualifies this condition because:

$$\bar{p} \geq \frac{w_0}{\mu(v)(1 - \underline{\rho})} + w_0 > \frac{w_0}{\mu(v - \epsilon)(1 - \underline{\rho})} + w_0$$

The right-most inequality is by the monotonicity of function  $\mu(v)$ . Then it must be true that

$$\pi'_{-i} \geq \mu(v - \epsilon)(v - \epsilon - \bar{p}) \left(1 - \frac{w_0}{\mu(v - \epsilon)(\bar{p} - w_0)}\right)$$

As  $\epsilon$  goes to zero, the right-hand side can be arbitrarily close to the total profit  $\pi$  in Eq. (56). Since firm  $-i$ 's profit before the deviation is strictly lower than  $\pi$ , i.e.  $\pi_{-i} < \pi$ , there must exist  $\epsilon > 0$  small enough, such that

$$\pi'_{-i} > \pi_{-i}$$

Therefore, firm  $-i$  can be strictly better off by undercutting firm  $i$ 's price, making firm  $i$ 's demand to be zero.  $\square$

Next, we show that all firms have the job completion rate  $\rho_i = \underline{\rho}$  in equilibrium. Formally, we introduce the following lemma, which also proves Proposition 3:

**Lemma 2.** *Under perfect competition, in a two-firm setting, any Nash equilibrium must satisfy  $\rho_i = \underline{\rho}$  for  $i = 1, 2$ . When  $\underline{\rho} = 0$ , it means any Nash equilibrium must satisfy  $\rho_i = 0$ , implying that  $\lambda = 0$  in equilibrium.*

*Proof.* Proof. Suppose there exists an equilibrium where  $\rho_i > \underline{\rho}$  for firm  $i$ . By Lemma 1, it must be true that firm  $i$  does not make any profit, i.e.  $v_i = p_i$ .

If firm  $i$ 's opponent, firm  $-i$ , does not participate in this equilibrium, then firm  $-i$  can earn a positive profit by choosing  $p_{-i} = p_i - 2\epsilon$ ,  $v_{-i} = v_i - \epsilon = p_i - \epsilon$ . By doing this, firm  $-i$  not only has a positive profit but also has a positive demand. To see why, note that when firm  $i$  is the only participating firm, it must be true that

$$\rho_i = 1 - \frac{w_0}{\mu(v_i)(v_i - w_0)} > \underline{\rho} \quad (65)$$

When firm  $-i$  reduces both its price and worker pay from the firm  $i$ 's level, its job completion rate is given by

$$\rho'_{-i} = 1 - \frac{w_0}{\mu(v_i - \epsilon)(v_i - 2\epsilon - w_0)} \quad (66)$$

which can be arbitrarily close to  $\rho_i$  as  $\epsilon$  goes to zero. Since the inequality of  $\rho_i > \underline{\rho}$  is strict, there exists  $\epsilon > 0$  such that firm  $-i$ 's job completion rate  $\rho_{-i} > \underline{\rho}$ . Thus, firm  $-i$  will be strictly better off by joining the gig economy.

The other possibility is that firm  $-i$  participates in the gig economy and has the same price, worker pay, demand, and job completion rate as firm  $i$ ; that is,

$$v_i = v_{-i} = p_i = p_{-i} = v, \mu_i = \mu_{-i} = \mu(v)/2, \rho_i = \rho_{-i} > \underline{\rho}$$

Then firm  $-i$  can also be strictly better off by choosing  $v'_{-i} = v - \epsilon$  and  $p'_{-i} = v - 2\epsilon$ . Firm  $-i$ 's job completion rate  $\rho'_{-i}$  has the same form as in Eq. (66) and can be arbitrarily close to  $\rho_i$  as  $\epsilon$  approaches zero. Therefore, firm  $-i$  can make a positive profit from the deviation.

Therefore, to conclude, there cannot exist a Nash equilibrium with  $\rho_i > \underline{\rho}$  for any  $i$ .  $\square$   $\square$

## C.2 Imperfect competition (Proof of Proposition 4)

### Proof of Proposition 4

*Proof.* Proof. We prove the two parts of the proposition separately.

**Part 1: Market Formation ( $\lambda^* > 0$ ).** We prove the statement by contradiction. Suppose there exists an equilibrium with  $\lambda^* = 0$  (no market formation), and at the same time, there exists a firm  $i$  that satisfies Eq. (15) for any pricing strategies  $\mathbf{v}_{-i}$  chosen by other firms.

Consider firm  $i$  setting a price  $v'_i$  such that Eq. (15) holds. By definition, this implies  $v'_i > w_0$ , and the firm achieves a size  $s_i > w_0$ . Under this condition, by Theorem 2, Item 3, firm  $i$  alone can sustain a positive worker arrival rate ( $\lambda^* > 0$ ) and operate profitably as the only participating firm.

This contradicts the assumption that  $\lambda^* = 0$ , as firm  $i$  has a clear incentive to participate in the gig economy and create a market. Therefore,  $\lambda^* = 0$  is impossible, and it must hold that  $\lambda^* > 0$ .

**Part 2: Participation of Other Firms.** Given that  $\lambda^* > 0$  holds in the worker pay subgame, consider any other firm  $j \neq i$  that satisfies  $s_j = (v_j - w_0)\mu_j(v_j; \mathbf{v}_{-j}) > 0$  for any pricing strategies  $\mathbf{v}_{-j}$  chosen by other firms. Such a firm  $j$  can set its pay rate to  $w_0$  and participate in the gig economy.

Since  $\lambda^* > 0$ , the shared worker pool is non-empty, and firm  $j$  will have positive demand. With  $s_j > 0$ , firm  $j$  can achieve positive profits by participating, completing the proof of the second part of the proposition.  $\square$

### C.3 Independent demand (Proof of Proposition 5)

*Proof.* Proof. The goal of the proof is to demonstrate that, given the sizes of other firms,  $\mathbf{s}_{-i}$ , it is a (weakly) dominant strategy for firm  $i$  to maximize its own size  $s_i$ , i.e., by choosing the price  $v_i$  such that  $s_i^* = \bar{s}_i$ .

The structure of the proof is as follows: We first introduce a more general statement, Proposition 8, which states that, all else being equal, the profit of firm  $i$  in equilibrium is non-decreasing with respect to the size of firm  $j$  (including when  $i = j$ ). Since this is a stronger statement than Proposition 5, and Proposition 5 is implied by it, we directly prove Proposition 8.

**Proposition 8.** *Consider an equilibrium  $(\mathbf{v}^*, \boldsymbol{\mu}^*, \mathbf{p}^*, \lambda^*)$  described in Theorem 1. Suppose each firm's size only depends on its own price. Then for any two firms  $i, j$ , all else being equal, the equilibrium profit of firm  $i$ ,  $\pi_i^*$ , is a non-decreasing function of the size of firm  $j$ ,  $s_j^*$ . Furthermore,  $\pi_i^*$  is also a non-decreasing function of firm  $i$ 's own size,  $s_i^*$ .*

### Proof of Proposition 8

To prove this result, we leverage an intermediate result presented in Lemma 3.

**Lemma 3.** *The equilibrium job completion rate described in Eq. (37) in Proposition 7 is non-decreasing in any firm's size.*

*Proof.* Proof of Lemma 3 Let the right-hand side of Eq. (37) be denoted as  $\text{RHS}(s_i, \rho)$ . Let  $\rho$  and  $\rho'$  represent the equilibrium job completion rates calculated using Eq. (37), with firm  $i$ 's size being  $s_i$  and  $s'_i$ , respectively. Then:

$$1 - \rho = \frac{w_0}{\sum_{j \neq i} (s_j - \frac{w_0 \rho}{(1-\rho)^2})^+ + (s_i - \frac{w_0 \rho}{(1-\rho)^2})^+} := \text{RHS}(s_i, \rho), \quad (67)$$

$$1 - \rho' = \frac{w_0}{\sum_{j \neq i} (s_j - \frac{w_0 \rho'}{(1-\rho')^2})^+ + (s'_i - \frac{w_0 \rho'}{(1-\rho')^2})^+} := \text{RHS}(s'_i, \rho'). \quad (68)$$

We prove by contradiction and assume that  $\rho$  is strictly decreasing in  $s_i$ , i.e.,  $s_i < s'_i$  and  $\rho > \rho'$ . The proof relies on the following two key statements:

1. Fixing  $\rho$ ,  $\text{RHS}(s_i, \rho)$  is weakly decreasing in  $s_i$ .
2. Fixing  $s_i$ ,  $\text{RHS}(s_i, \rho)$  is weakly increasing in  $\rho$ .

Both statements can be verified as follows: - Since  $s_i$  appears in the denominator and all terms are positive,  $\text{RHS}(s_i, \rho)$  is weakly decreasing in  $s_i$ . - Since  $\text{RHS}(s_i, \rho)$  is weakly decreasing in  $-\frac{w_0 \rho}{(1-\rho)^2}$ , and  $-\frac{w_0 \rho}{(1-\rho)^2}$  is strictly decreasing in  $\rho$ , it follows that  $\text{RHS}(s_i, \rho)$  is weakly increasing in  $\rho$ .

By the assumption that  $s_i < s'_i$  and  $\rho > \rho'$ , we have:

$$\text{RHS}(s'_i, \rho') \leq \text{RHS}(s'_i, \rho) \leq \text{RHS}(s_i, \rho).$$

This implies:

$$1 - \rho' \leq 1 - \rho,$$

contradicting the assumption that  $\rho' < \rho$ . Thus,  $\rho^*(s_i; s_{-i})$  must be non-decreasing in  $s_i$ .  $\square$

*Proof.* Proof of Proposition 8 By Proposition 7, the market formation equilibrium is uniquely defined by Eq. (37) and Eq. (38), and the equilibrium profit of firm  $i$  is given by:

$$\pi_i^* = r_i^* \rho^*.$$

We show that  $\pi_i^*$  is non-decreasing in the size of any firm  $j$ .

Using Eq. (37) and Eq. (38),  $\pi_i^*$  can be expressed as:

$$\pi_i^*(s_1, s_2, \dots, s_N) = r_i^*(\rho^*, s_i) \cdot \rho^* = r_i^*(\rho^*(s_1, s_2, \dots, s_N), s_i) \cdot \rho^*(s_1, s_2, \dots, s_N). \quad (69)$$

Taking the derivative of  $\pi_i^*$  with respect to  $s_j$ , we have:

$$\frac{\partial \pi_i^*}{\partial s_j} = \frac{\partial r_i^*(\rho^*, s_i)}{\partial s_j} \cdot \rho^* + r_i^* \cdot \frac{\partial \rho^*}{\partial s_j}. \quad (70)$$

We verify that all terms in Eq. (70) are non-negative: - By Item 2 in Proposition 7,  $\rho^* > 0$  and  $r_i^* > 0$ . - By Lemma 3,  $\frac{\partial \rho^*}{\partial s_j} > 0$ .

The only remaining term is  $\frac{\partial r_i^*(\rho^*, s_i)}{\partial s_j}$ , which depends on whether  $j = i$ . We analyze both cases:

1. When  $j = i$ :

$$\frac{\partial r_i^*(\rho^*, s_i)}{\partial s_i} = \begin{cases} \frac{\partial r_i^*}{\partial \rho^*} \cdot \frac{\partial \rho^*}{\partial s_i}, & \text{if } \frac{w_0 \rho^*}{(1-\rho^*)^2} < s_i, \\ 1, & \text{otherwise.} \end{cases} \quad (71)$$

When  $\frac{w_0 \rho^*}{(1-\rho^*)^2} < s_i$ ,  $r_i^* = \frac{w_0 \rho^*}{(1-\rho^*)^2}$ , which is clearly increasing in  $\rho^*$ . Since  $\rho^*$  is non-decreasing in  $s_i$ ,  $\frac{\partial r_i^*(\rho^*, s_i)}{\partial s_i} \geq 0$ .

2. When  $j \neq i$ :

$$\frac{\partial r_i^*(\rho^*, s_i)}{\partial s_j} = \begin{cases} \frac{\partial r_i^*}{\partial \rho^*} \cdot \frac{\partial \rho^*}{\partial s_j}, & \text{if } \frac{w_0 \rho^*}{(1-\rho^*)^2} < s_i, \\ 0, & \text{otherwise.} \end{cases} \quad (72)$$

Thus,  $\frac{\partial r_i^*(\rho^*, s_i)}{\partial s_j} \geq 0$ .

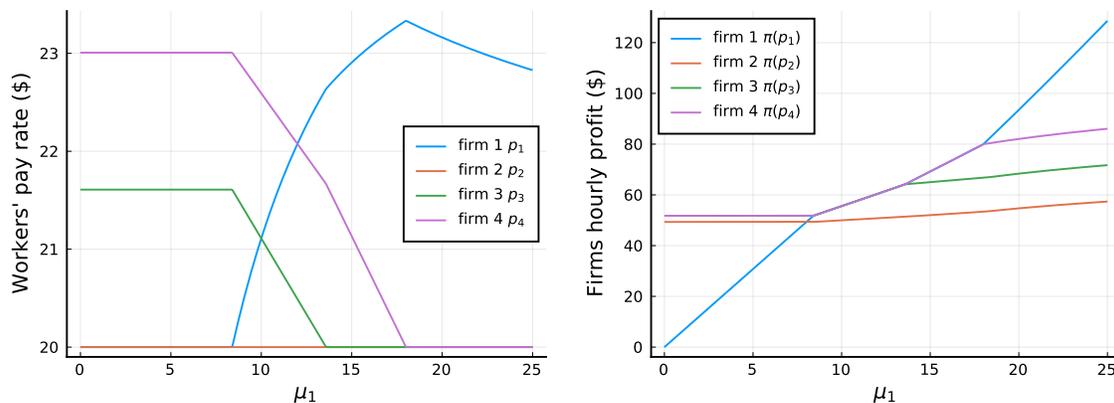
Combining these results, we have:

$$\frac{\partial \pi_i^*}{\partial s_j} \geq 0.$$

Therefore, the equilibrium profit of firm  $i$  is non-decreasing in the size of any firm  $j$ .  $\square$   $\square$

$\square$

## D Additional numerical examples



(a) Workers' pay rates  $p_1, p_2, p_3, p_4$  at the Nash equilibrium.

(b) Firms' hourly profit  $\pi(p_1), \pi(p_2), \pi(p_3), \pi(p_4)$  at the Nash equilibrium.

Figure 10: Equilibria in a setting with four firms when the first one is growing. The parameters are the same as in Figure 2, except for the job arrival rates. Here, we have  $\mu_2 = 8, \mu_3 = 10, \mu_4 = 12$ , and we vary  $\mu_1$  from 0 to 25.

**Four firms** Figure 10 illustrates an example with four firms. In this more complex setting, we examine the impact of a growing firm (by varying  $\mu_1$ ) on the equilibrium prices and profit, keeping the other firms' constant ( $\mu_2, \mu_3, \mu_4 = 8, 10, 12$ ). If we first look at Figure 10a, we see that when  $\mu_1$  is small, firms 1 and 2 pay  $w_0$  while firms 3 and 4 have higher pay rates (and therefore the same hourly profit). As firm 1 grows, it starts to pay workers more, which allows firms 3 and 4 to pay less until they can also free ride and pay  $w_0$ .

Figure 10b shows the resulting profit outcomes. Note the profits of firms 2, 3, and 4 stay constant as long as firm 1 is small enough to pay  $w_0$ , but increase when firm 1 starts to pay above  $w_0$  and begins contributing to increasing the size of the worker pool, which benefits all firms. Interestingly, while firm 1's profit grows rapidly at first with its job rate  $\mu_1$ , this growth is slowed down once it becomes big enough to allow the other firms to "free ride" and pay less. After a "phase transition", in which firm 1 is big enough to be the only firm that pays above the minimum rate and all other firms are free-riding, the growth of firm 1's profit recovers as the other firms cannot increase their marginal profit further.

## E Data source for figures and charts

- India: the market share data for leading companies are according to this Quartz article: *As Uber sputters, Ola is really stepping on the gas in India*, Quartz, <https://qz.com/india/1200878/with-uber-in-crisis-ola-zooms-ahead-in-indias-taxi-wars/>. References for the market entry year for each company is listed in the "Source" column below.

Table 3: Share of ride hailing market across India as of December 2017 and leading companies' year of market entry, by company.

	Market share	Market entry time	Source
Ola	56.2%	Dec. 2010	Crunchbase
Uber	39.6%	Aug. 2013	Uber blog
Jugnoo	2.5%	Nov. 2014	Crunchbase
Meru	0.8%	2014*	Economic Times
ixigo Cabs	0.9%	July 2015	startupleadership.com

*Note:* 2014 is the year Meru introduced app-based ride-hailing service; the company was founded much earlier in 2007 as a call-based taxi company.

- Indonesia: the market share data for leading companies are according to this article: *The market share of Gojek is almost 80 percent*, industryco.id, <http://en.industry.co.id/read/6771/the-market-share-of-gojek-is-almost-80-percent>. References for the market entry year for each company is listed in the "Source" column below.

Table 4: Market share of the ride-hailing transportation industry in Indonesia and their year of market entry as of April 2018, by company

	Market share	Market entry time	Source
Go-Jek	79.2%	2015*	Reuters
Grab	14.69%	June 2012	Crunchbase
Uber*	6.11%	N/A	N/A

*Note:* 2015 is the year Go-jek introduced app-based ride-hailing services; the company was founded in 2009. The time when Uber entered Indonesia market is not available; It exit Indonesia market in April 2018.

- China: the market share data for leading companies are according to this report: *China internet report 2019*, page 58, south China Morning Post, [https://multimedia.scmp.com/infographics/china-internet/pdf/china\\_internet\\_report\\_2019.pdf](https://multimedia.scmp.com/infographics/china-internet/pdf/china_internet_report_2019.pdf). References for the market entry year for each company is listed in the “Source” column below.

Table 5: Market share of ride-sharing market in china in 4th quarter 2018 and the year of market entry, by company

	Market share	Market entry time	Source
Didi	91%	Sept. 2012	Crunchbase
Shouqi	2%	2015	Crunchbase
Meituan	2%	Feb. 2017	Financial Times
Shenzhou	2%	Jan. 2015	Owler
Caocao	2%	Nov. 2015	South China Morning Post
Yidao	1%	May 2010	Crunchbase

- Latin America: market shares are computed based on active user numbers for leading players by Techcrunch: Latin America is the next stage in the race for dominance in the ride-hailing market, Techcrunch, <https://techcrunch.com/2018/09/07/latin-america-is-the-next-stage-in-the-race-for-dominance-in-the-ride-hailing-market/>. References for the market entry year for each company is listed in the “Source” column below.

## F Simulations Details

### F.1 Simulation Parameters

Table 7 presents the parameters of our “base setting”. This means that, if a parameter is not explicitly mentioned in the main paper, it has been set as listed here. At first sight, the numbers here might seem low (e.g., we typically have a maximum of 4 customer arrival per

Table 6: Market share of ride-sharing market in latin America 2018 and the year of market entry, by company

	Market share	Market entry time	Source
Uber	57.14%	2013	Techcrunch
Didi/99	22.22%	2012*	Crunchbase
Cabify	20.63%	2012	CoMotion News

*Note:* Didi acquired 99 in 2018. Year 2012 is the time that 99 entered the Latin America market

hour). The goal is two-fold: this allow us to keep a reasonably low queue cap, which is what governs tractability. This is also a good model for spatial platforms such as ridesharing or food delivery. Indeed, an often-used model is to represent a city as a collection of small queues representing the drivers in each neighborhood. Therefore, the simulation does not represent the entire platform but rather its operation in a limited spatial location. On this thought, the parameters have also been chosen to match spatial platforms typical worker utilization (below 70%), and their high supply elasticity. In this set of parameters, the firms are also typically more demand-constrained than supply-constrained, and they have independent demand (e.g., Uber and Doordash).

## F.2 Impact of Dedicated Worker Pools

Figure 11 illustrates our dedicated worker pools setting, described in Section 5.3. Figure 12 then presents our simulation results in this setting. Recall that  $N_{\text{dedi-1}}, N_{\text{dedi-2}}$  are the maximum number of dedicated workers in the system (the size of the new worker pools), whereas  $N_{\text{multiapp}}$  is the maximum number of (potentially) multiapping workers. The x-axis varies the fraction of total supply that is dedicated, defined as  $(N_{\text{dedi-1}} + N_{\text{dedi-2}})/(N_{\text{dedi-1}} + N_{\text{dedi-2}} + N_{\text{multiapp}})$ , with the total supply held constant. The three columns vary the relative level of dedicated supply – for example, the second column has  $N_{\text{dedi-1}}/(N_{\text{dedi-1}} + N_{\text{dedi-2}}) = 25\%$ . The most striking result is that, in all plots, the dedicated supply is not utilized in equilibrium when the fraction of dedicated supply is below 80%, which is why the two firms have the same pay and profit. Indeed, multi-apping workers have lower wait times than dedicated workers because they have access to the two firms. When  $\alpha_{\text{supply}}$  and  $N_{\text{multiapp}}$  are high (large elastic pool), dedicated workers are kicked out of equilibrium because firms can get more multi-apping workers for a lower pay. However, this changes when the dedicated supply exceeds 80%. At this threshold, the limited availability of multi-apping workers forces firms to increase pay and use dedicated pools, which impacts profitability. Interestingly, the first column demonstrates that Firm 1, with no dedicated supply, can outperform Firm 2 due to a strong free-riding effect. Indeed, Firm 1 is “smaller”, as Firm 2 gets more demand once it starts to use dedicated workers. Nonetheless, Firm 1’s profit falls to zero without multi-apping workers. In short, dedicated

Parameter and description	Default value in simulations
$T_1$ — Average time of a task from Firm 1	1 hour
$T_2$ — Average time of a task from Firm 2	1 hour
$\alpha_{\text{supply}}$ — Supply sensitivity in the MNL model	5 (i.e., very high)
$N_{\text{multiapp}}$ — Maximum multi-apping workers	4 workers (typically more than enough)
$N_{\text{dedi-1}}$ — Maximum workers dedicated to firm 1	0 workers
$N_{\text{dedi-2}}$ — Maximum workers dedicated to firm 2	0 workers
$w_0$ — Workers' outside opportunity wage	\$40/hour (targets a 40%-70% utilization range)
$U_{\text{service}}$ — Service utility (same for both firms)	\$100
$U_{\text{no-service}}$ — Utility of a non-served customer	\$0
$D_{\text{dedi-1}}$ — Size of Firm 1's dedicated demand pool	2 customers per hour
$D_{\text{dedi-2}}$ — Size of Firm 2's dedicated demand pool	2 customers per hour
$D_{\text{multiapp}}$ — Size of the competitive demand pool	0 customers per hour
$\alpha_{\text{demand}}$ — Demand sensitivity in the MNL model	0.2 (i.e., a "nice" demand curve where firms can lower prices to get more demand smoothly)
Equilibrium concept	sequential 4-stage equilibrium as in Section 2.
Cap of the three queues	5 (lowest cap with < 2% probability of hitting the cap in all simulations).

Table 7: Default value of the simulation parameters, if otherwise specified.

workers are only useful if they are very few multi-apping workers are in the market. And free riding is present and very strong when one firm is "smaller" than another.

### F.3 Impact of Service Level Sensitivity

Figure 13 studies the impact of the service level sensitivity to service level by varying the parameter  $c_{\text{no-service}}$  in Eq. (19). We study three settings: full competition ( $D_{\text{dedi-1}} = D_{\text{dedi-2}} = 0, D_{\text{multiapp}} = 4$ ), no-competition and symmetric firms ( $D_{\text{multiapp}} = 0, D_{\text{dedi-1}} = D_{\text{dedi-2}} = 2$ ) and no-competition and asymmetric firms ( $D_{\text{multiapp}} = 0, D_{\text{dedi-1}} = 1, D_{\text{dedi-2}} = 3$ ). Here, the results are intuitive: looking at the middle column, we see that increasing the demand sensitivity to service level leads the firms to increase worker pay to increase the number of workers and, therefore, the service level. This, in turn, reduces the firm's profit until they leave the market. The third column shows that free-riding is still very strong. Interestingly, the firms make a higher total profit in equilibrium. The first column may be surprising at first sight. However, remember that competitive demand can choose

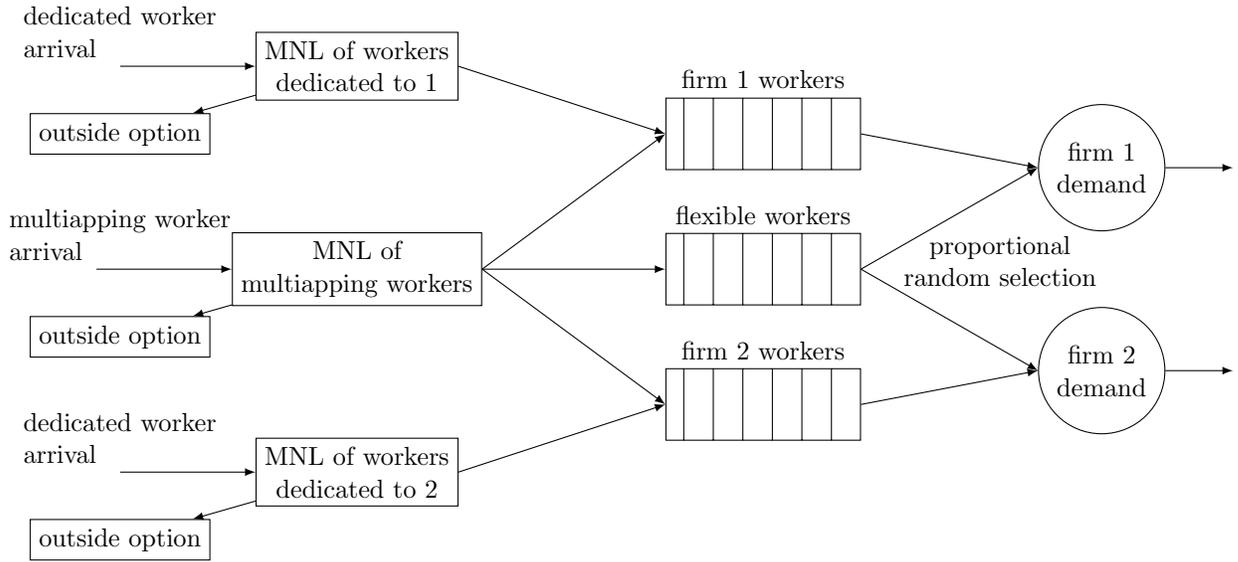


Figure 11: Queuing system with dedicated workers pools.

*Note:* This is the same as in Fig. 3, except that there are now three pools of workers, but only the multi-apping ones can join the three queues in the MNL model.

to be shared, which leads to much thicker markets with higher service levels. Because the equilibrium service level is much higher, increasing  $c_{\text{no-service}}$  in this limited way almost does not affect the equilibrium profit.

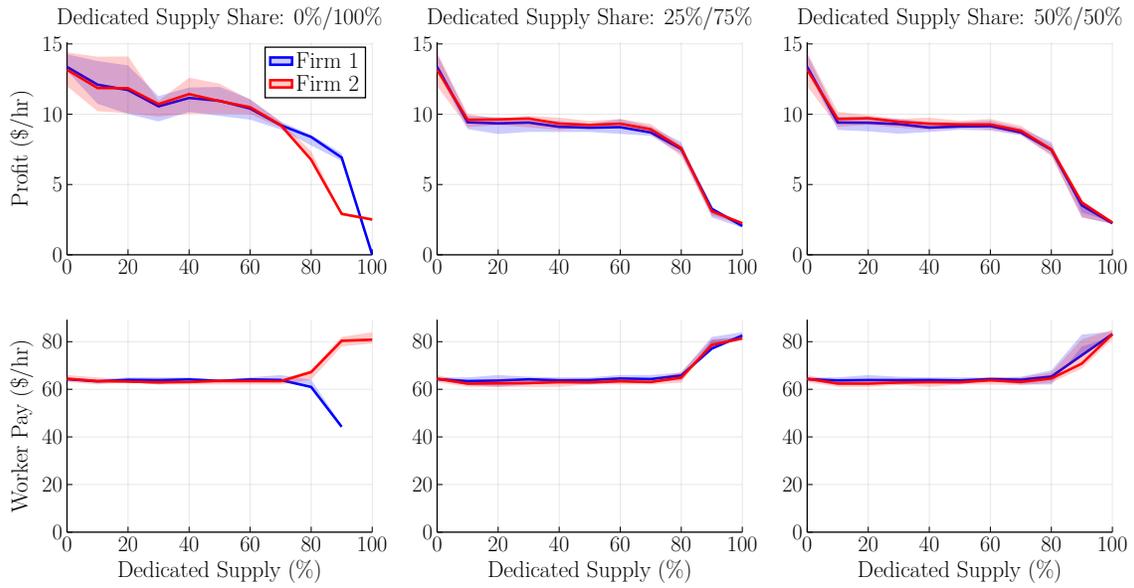


Figure 12: Impact of Private Labor Pools.

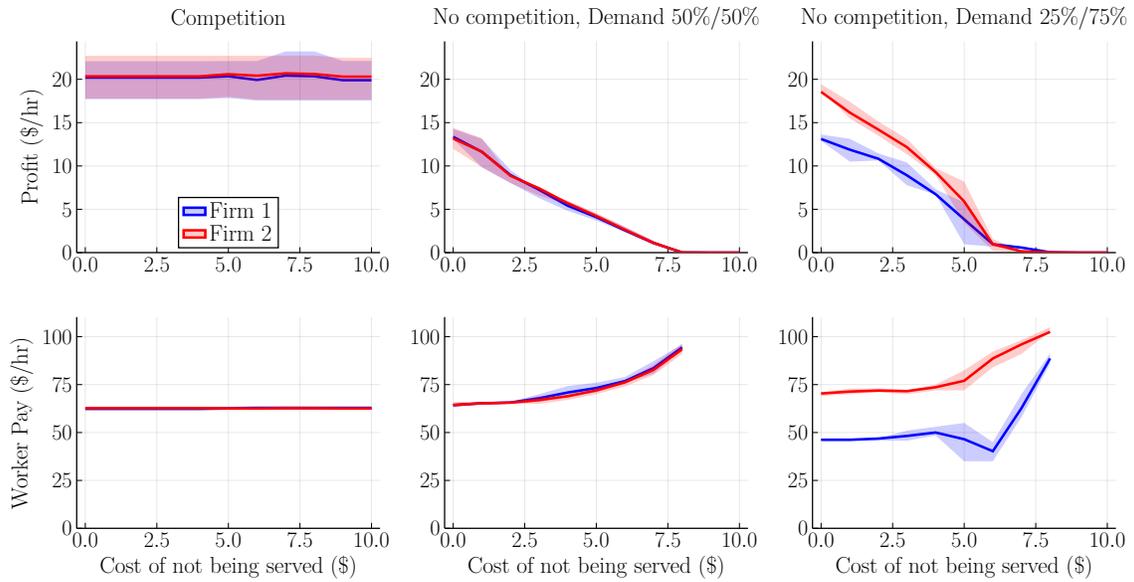


Figure 13: Impact of demand's sensitivity to service level.